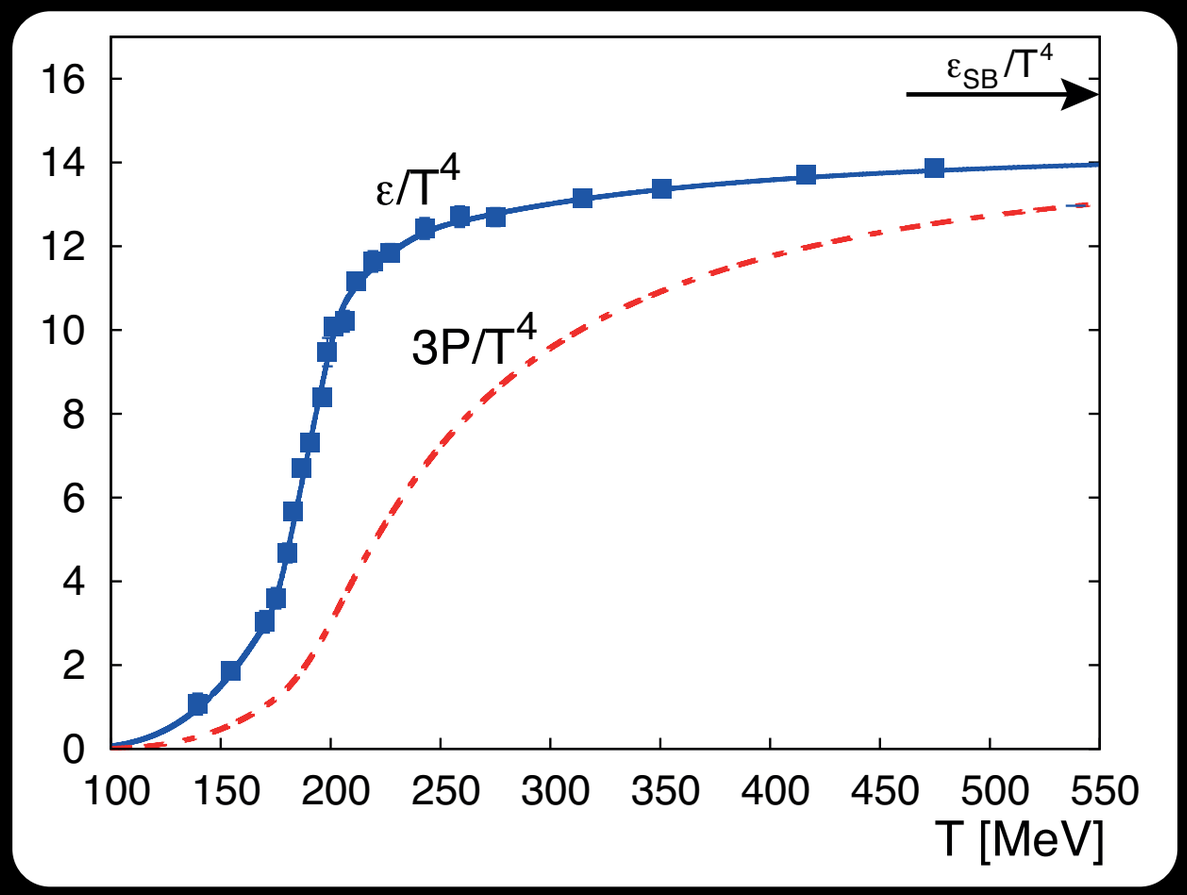
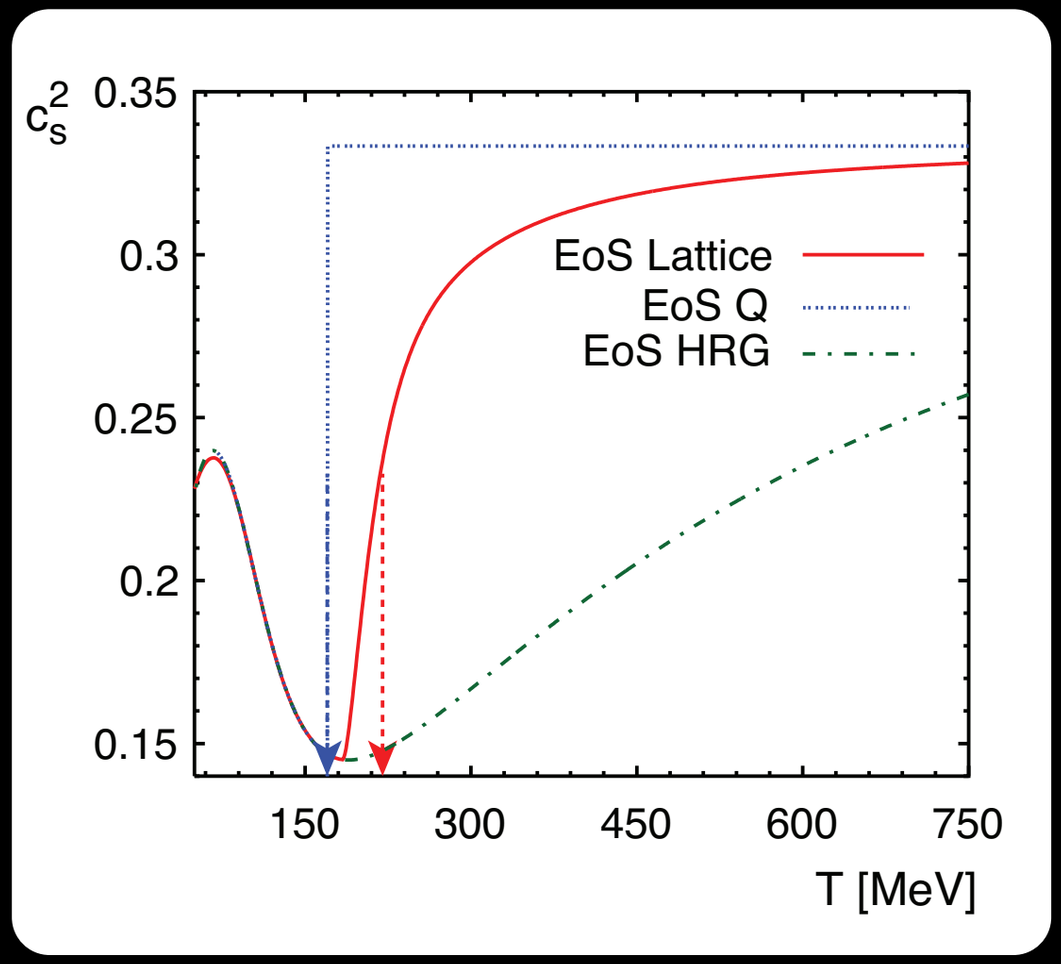
Velocity of Sound



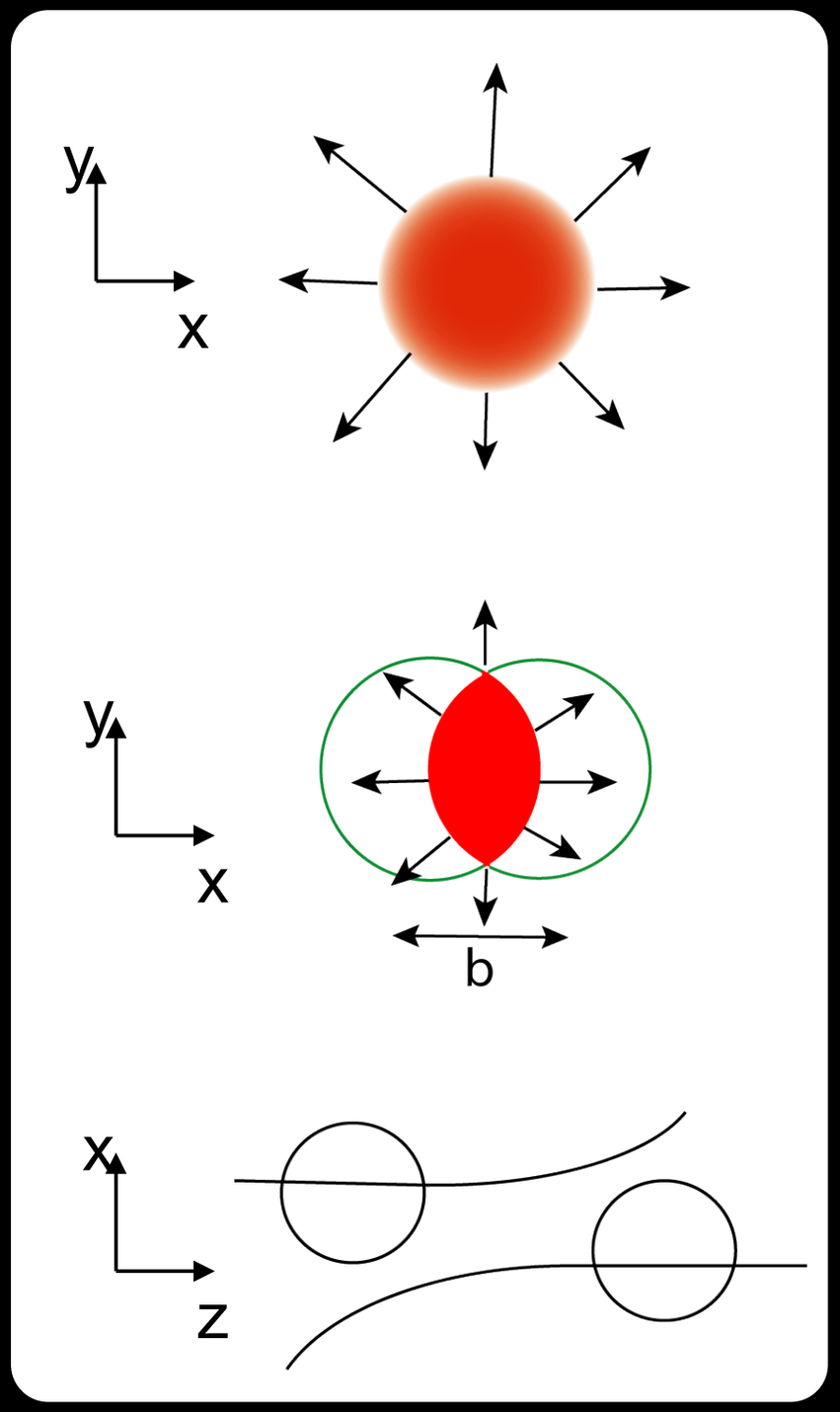


$$P_{\mathrm{QGP}} = \frac{1}{3}\varepsilon_{\mathrm{QGP}} = g\frac{\pi^2}{90}T^4$$

$$\text{velocity of sound } C_s = \sqrt{\frac{dP}{d\varepsilon}}$$

the magnitude of the collective motion is proportional to the velocity of sound

Collective Motion



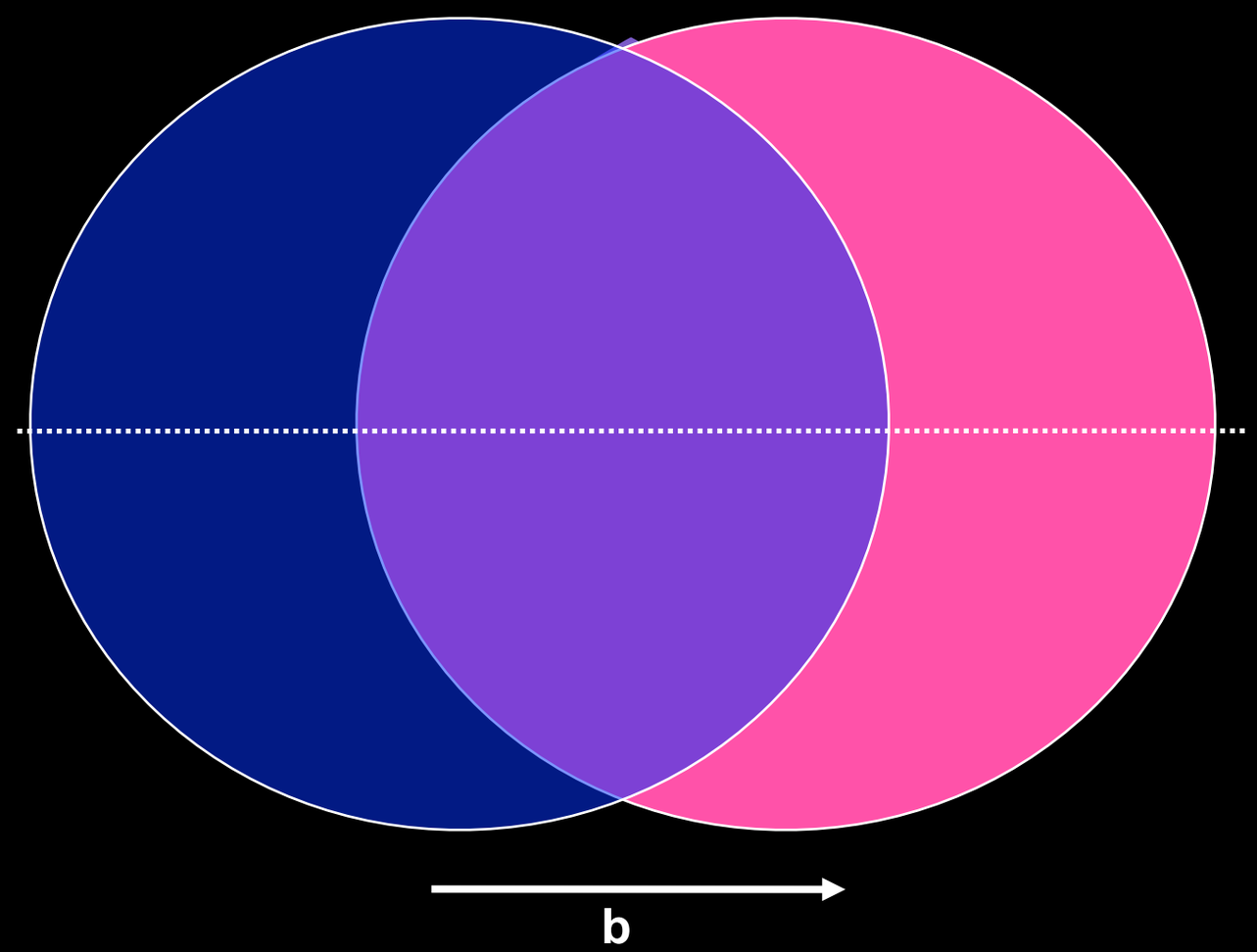
only type of transverse flow in central collision (b=0) is radial flow Integrates pressure history over complete expansion phase

elliptic flow (v_2) , v_4 , v_6, … caused by anisotropic initial overlap region (b > 0) more weight towards early stage of expansion

directed flow (v_1) , sensitive to earliest collision stage (b > 0), pre-equilibrium at forward rapidity, at midrapidity perhaps different origin

Elliptic Flow

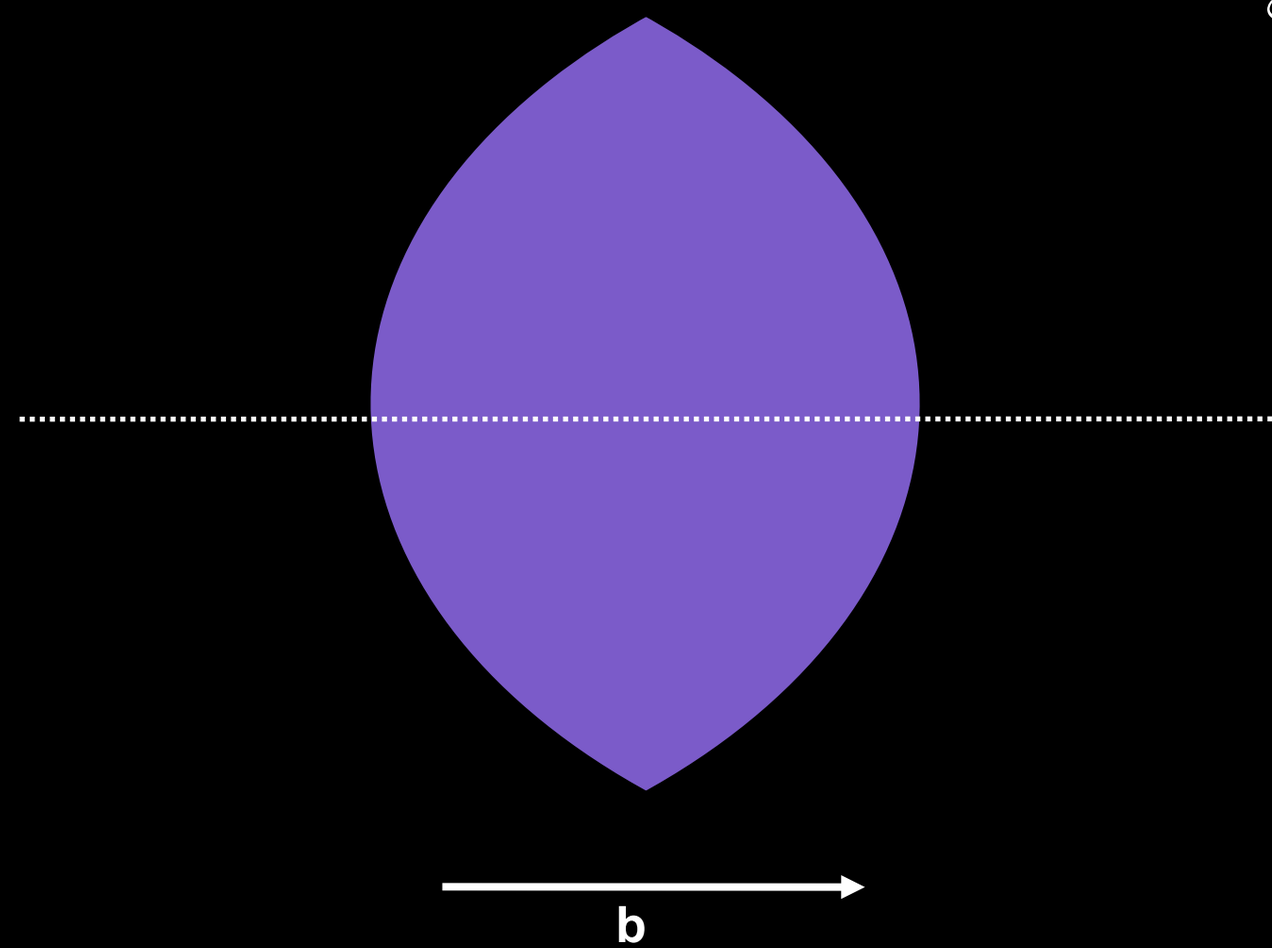
Animation: Mike Lisa



Elliptic Flow

Animation: Mike Lisa

$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$



Elliptic Flow

1) superposition of independent p+p:

Animation: Mike Lisa

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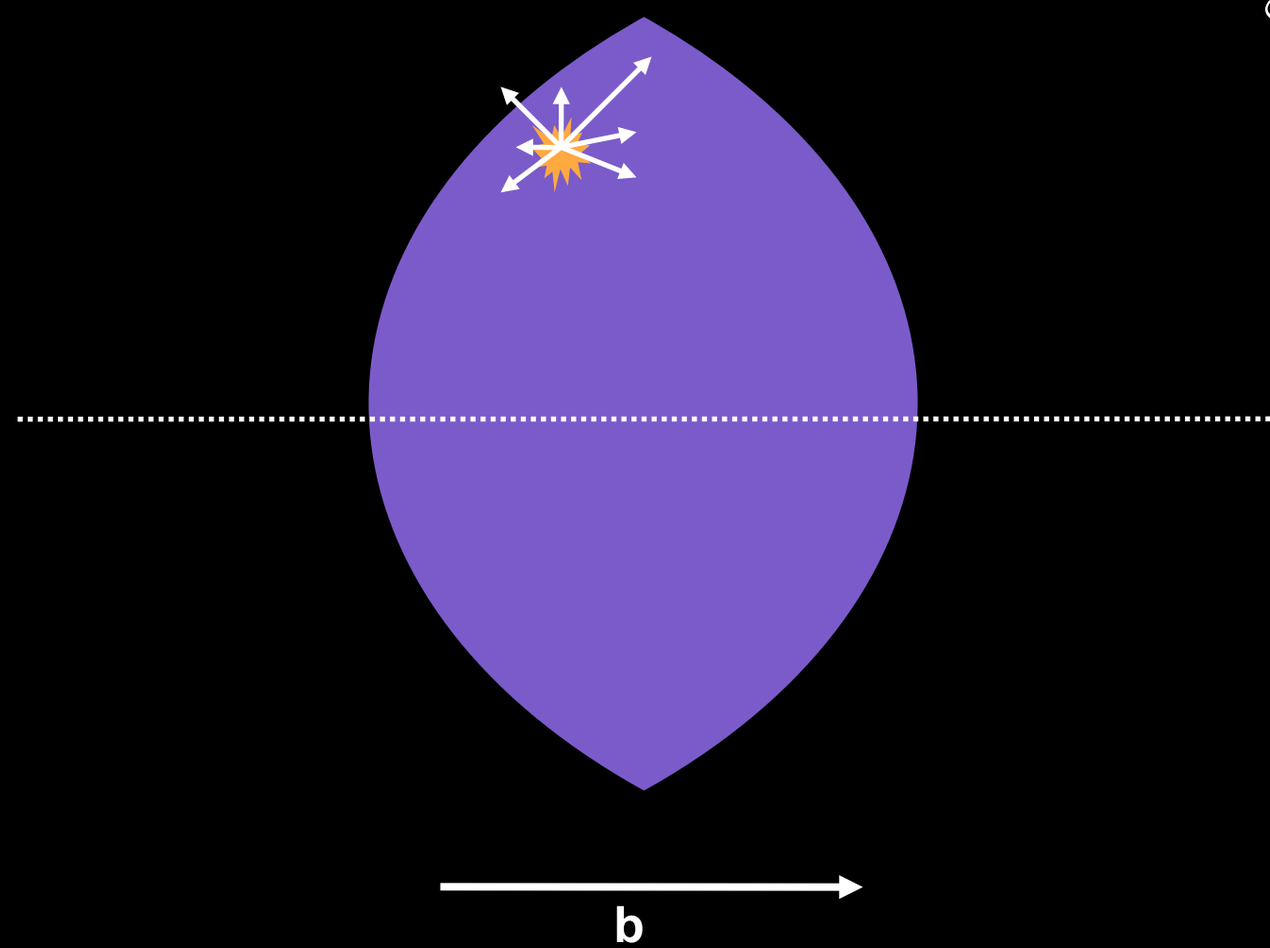

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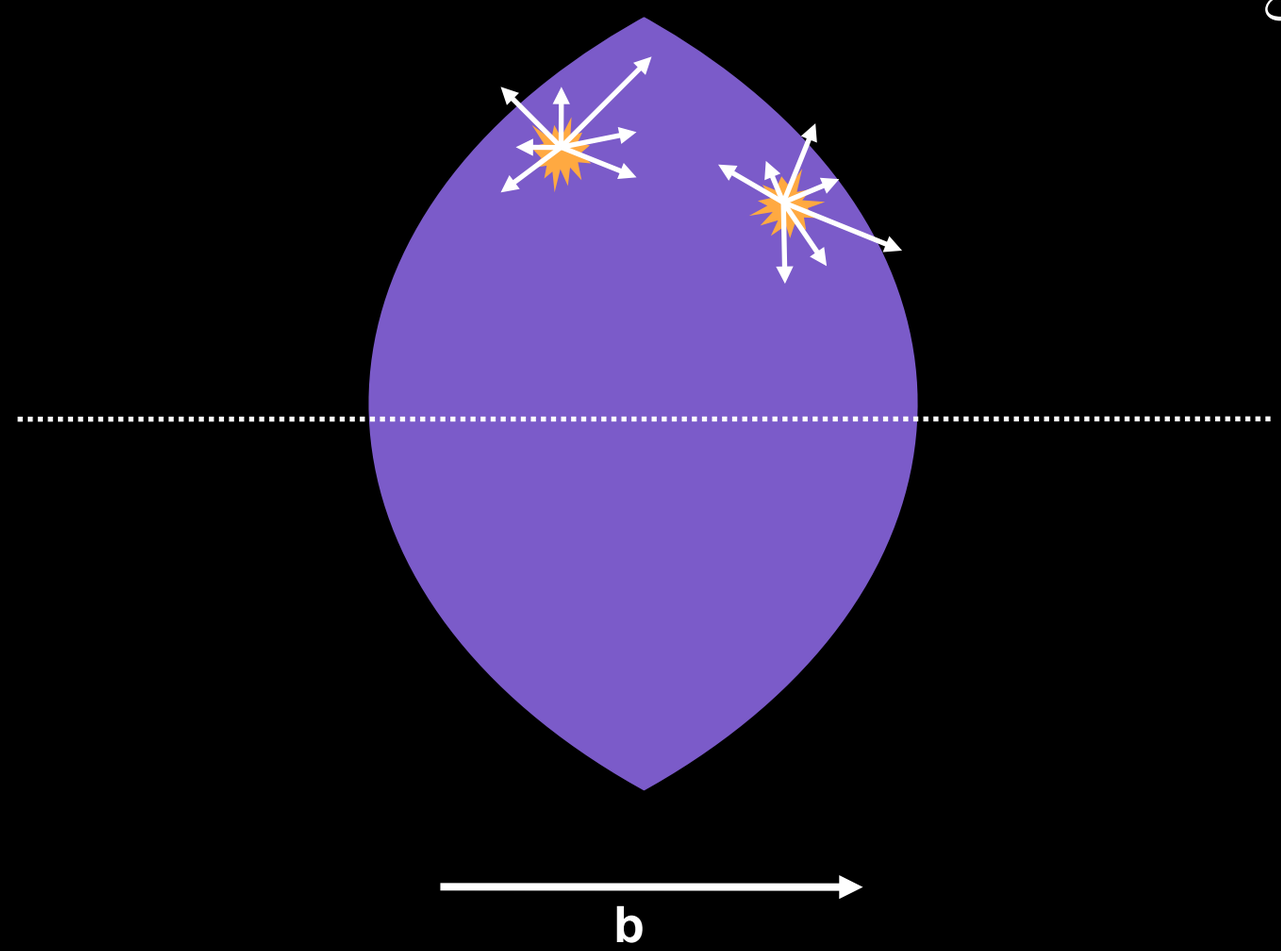

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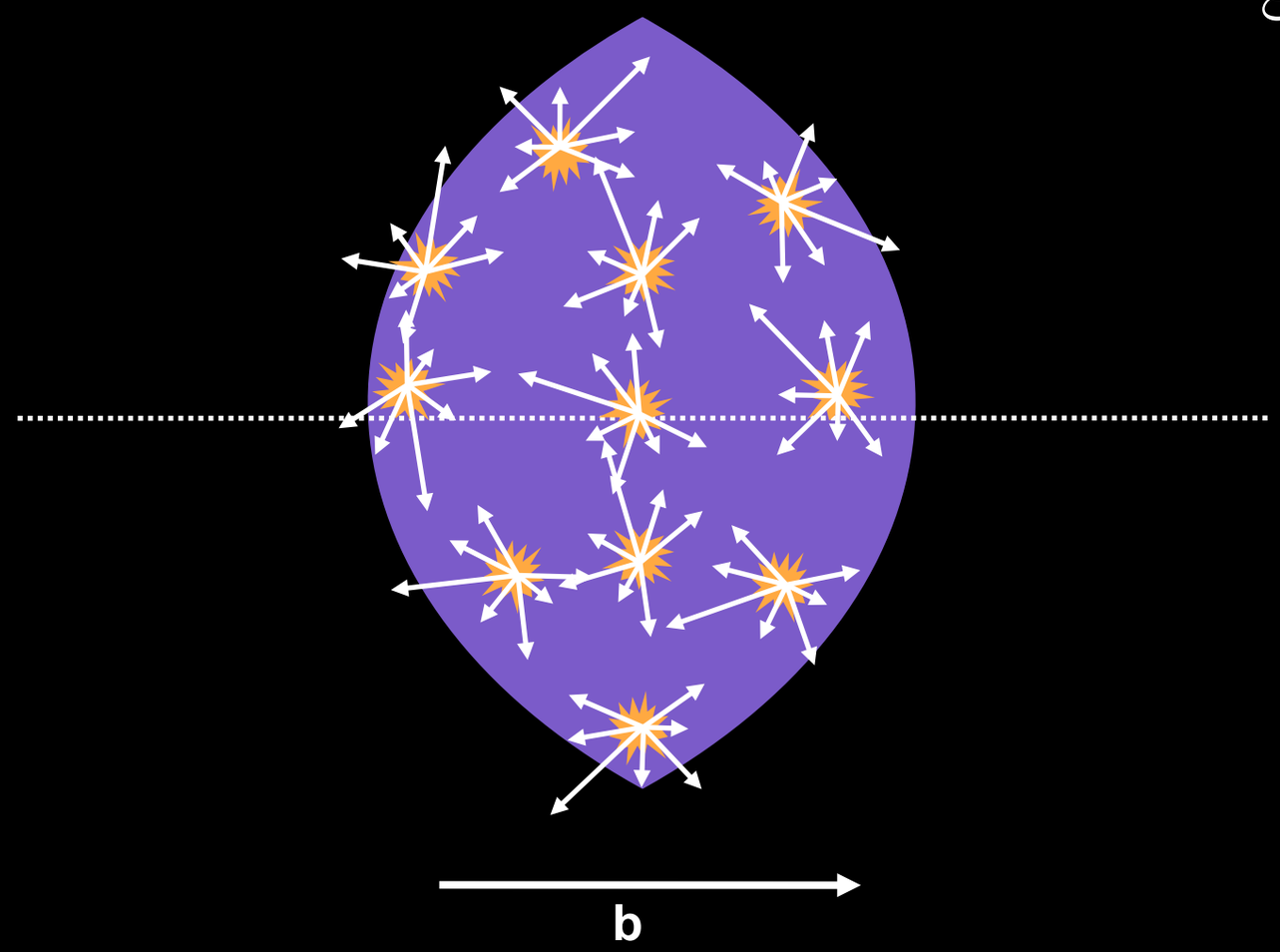


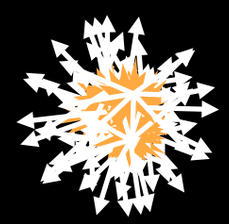
Elliptic Flow

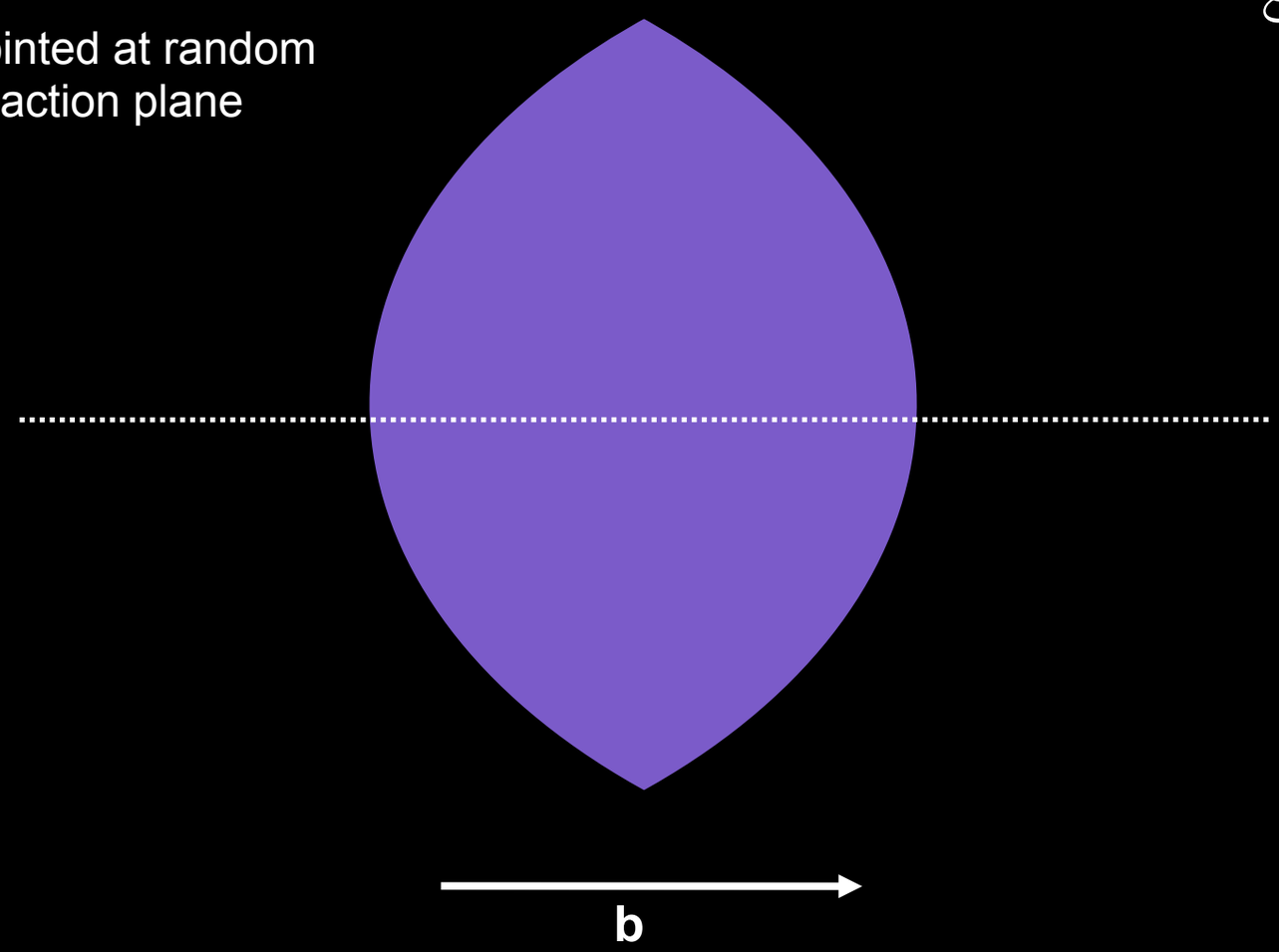
Animation: Mike Lisa

1) superposition of independent p+p:

momenta pointed at random
relative to reaction plane

$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$





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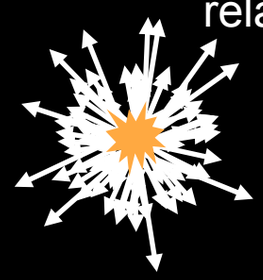
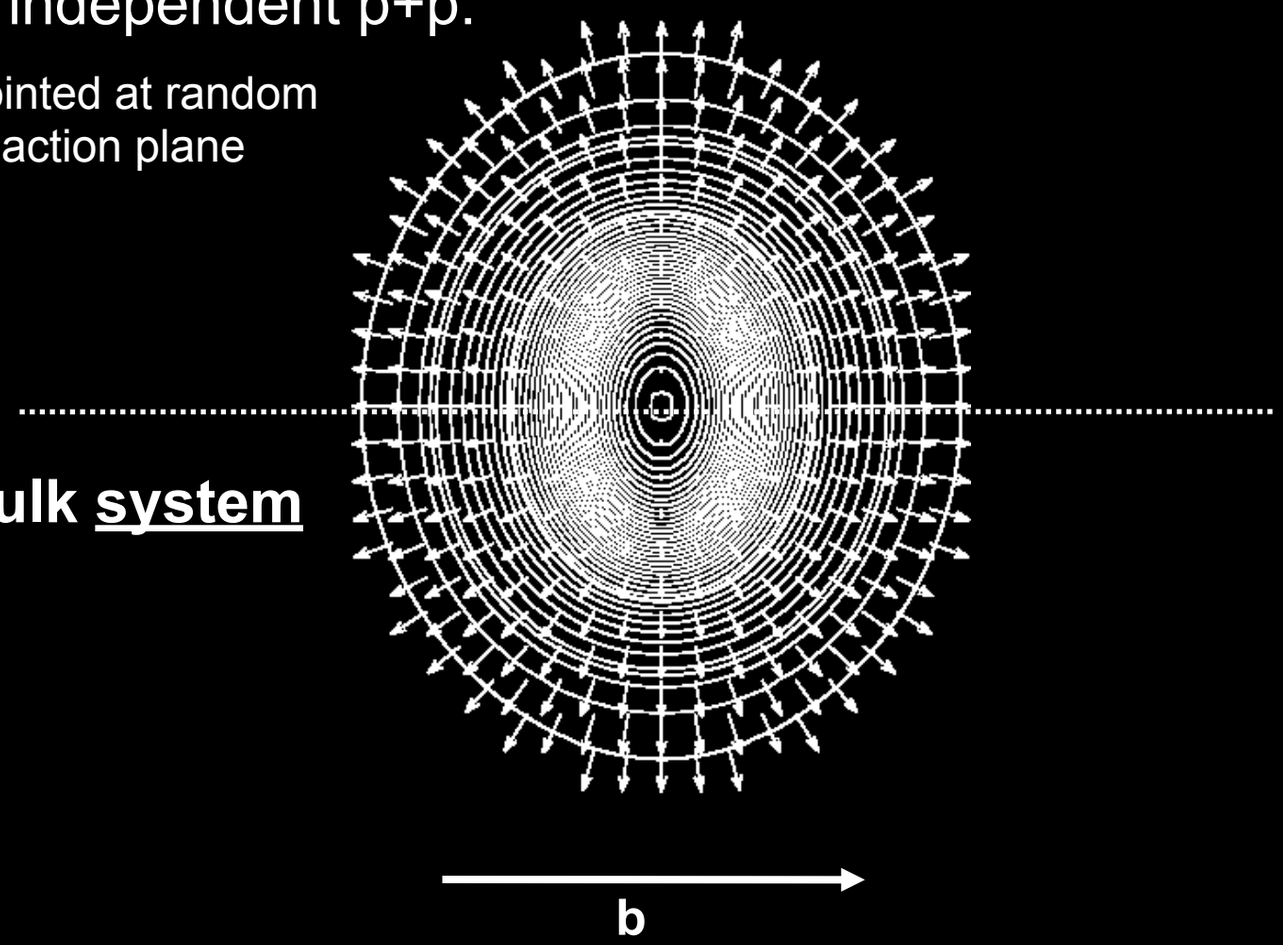


2) evolution as a **bulk <u>system</u>**



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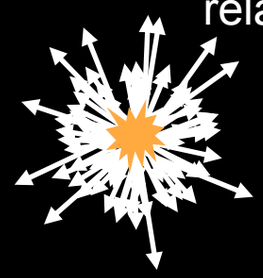

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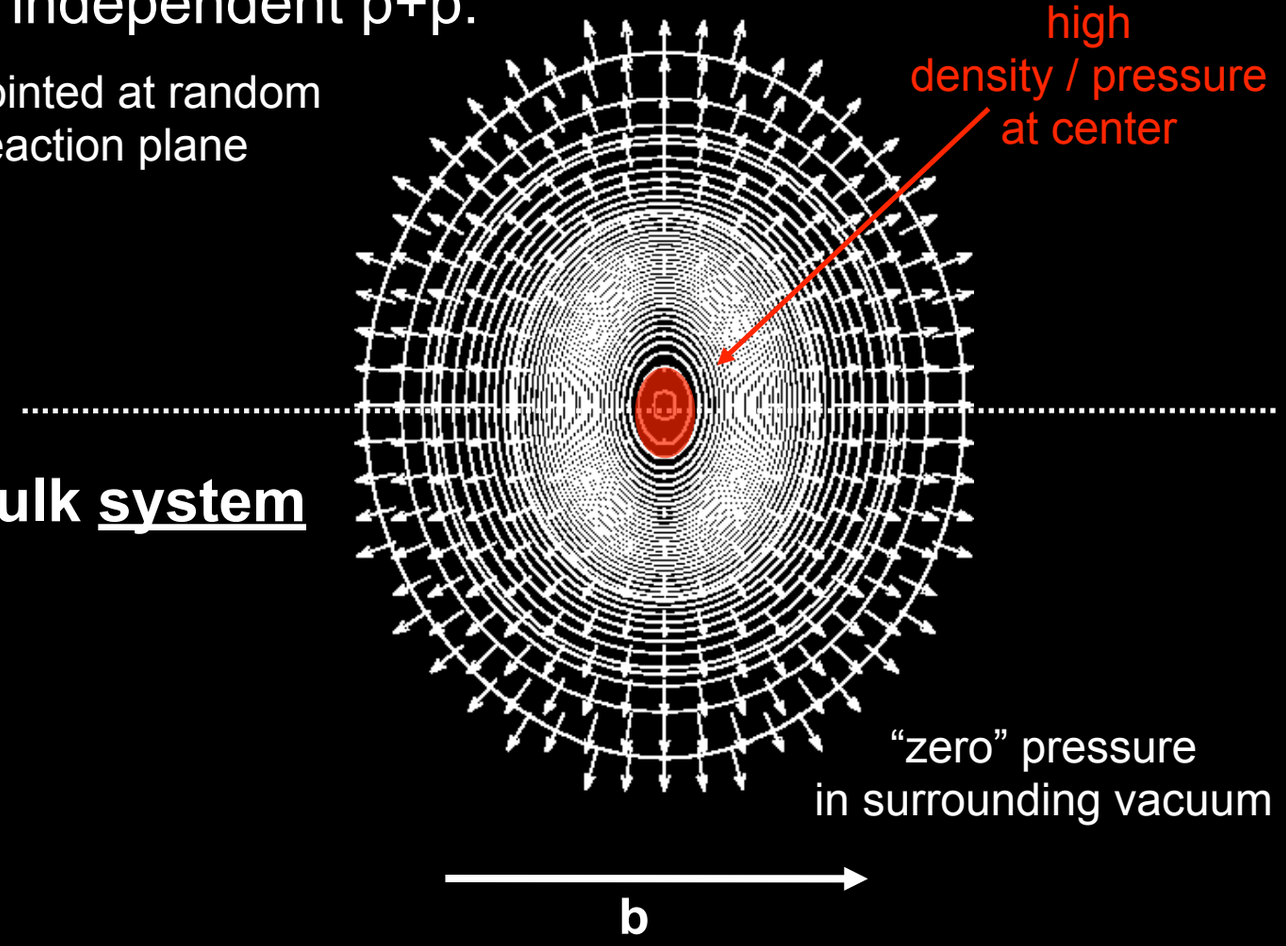

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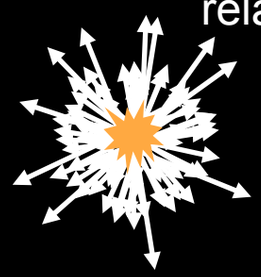

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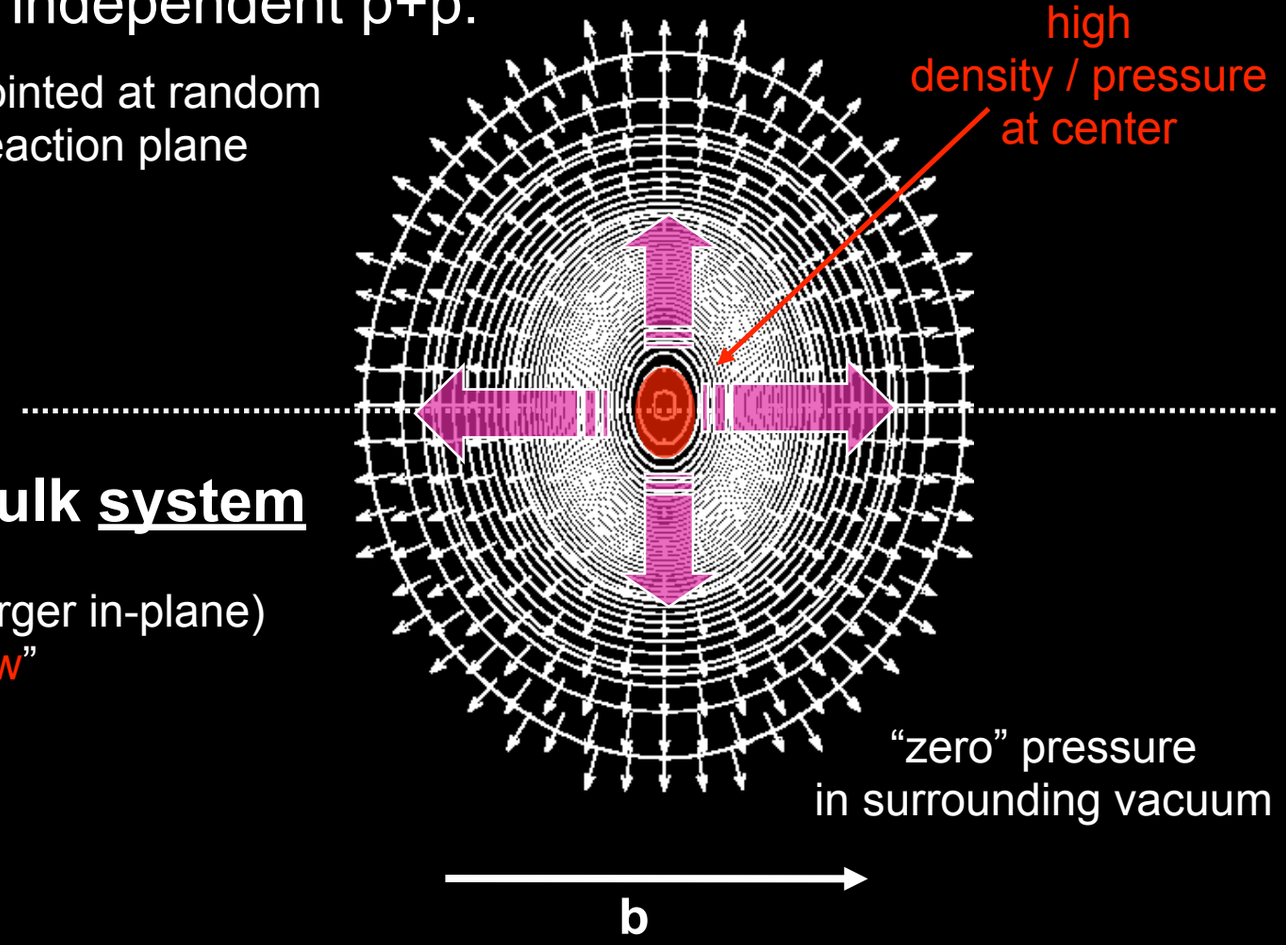
pressure gradients (larger in-plane) push bulk "out" → "flow"



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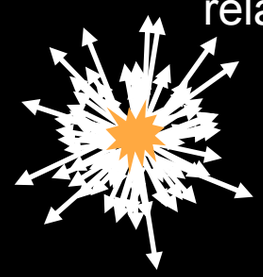

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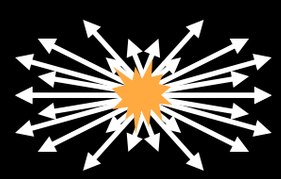
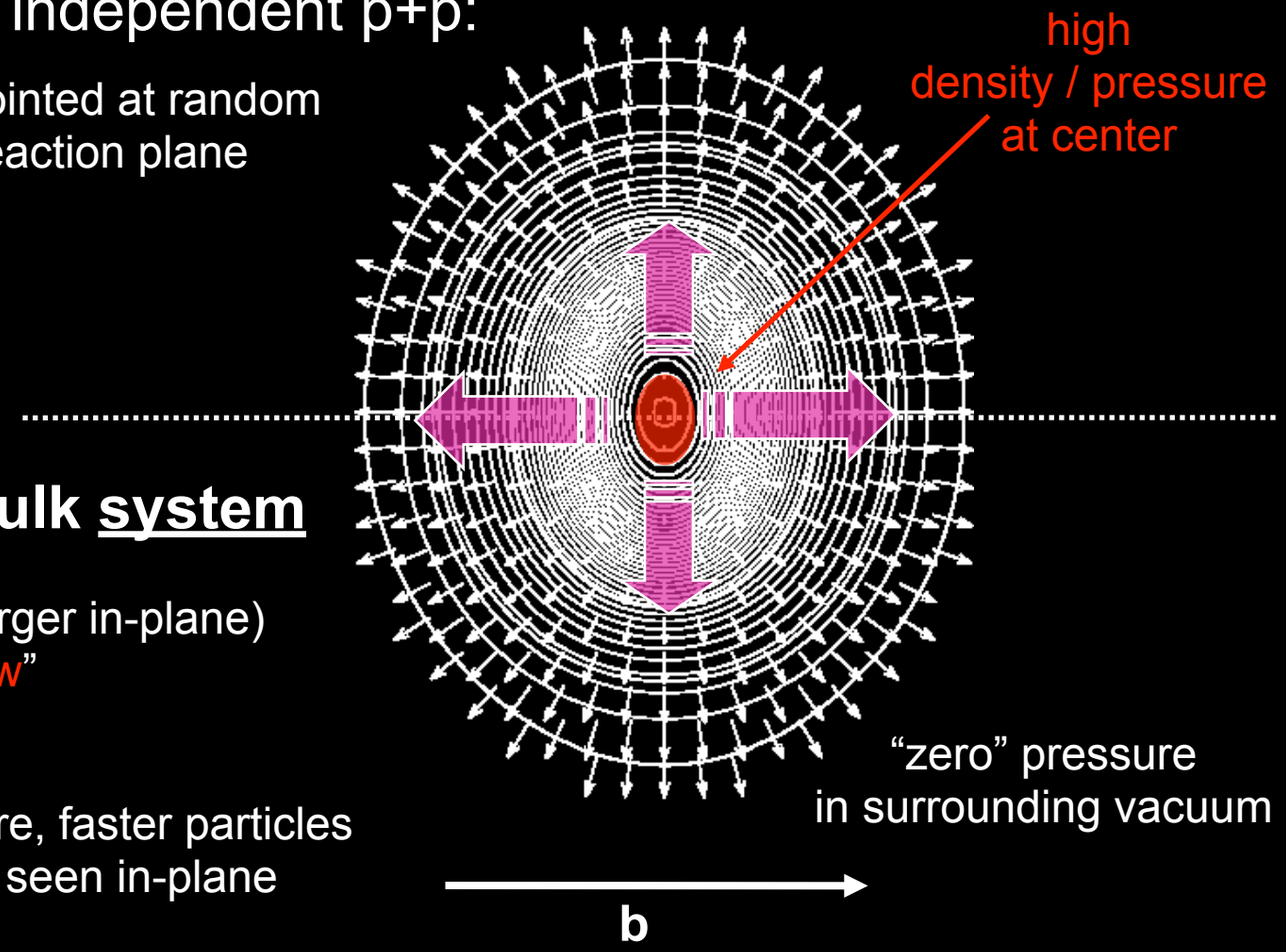


more, faster particles seen in-plane



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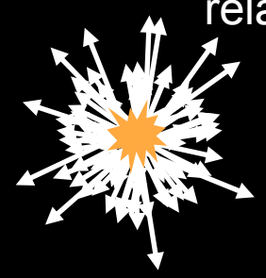

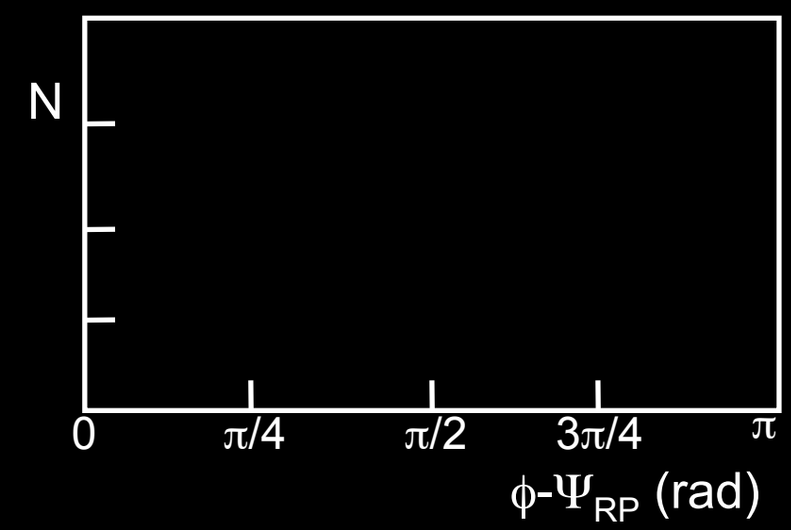

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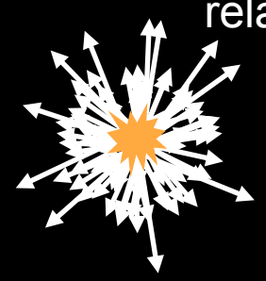

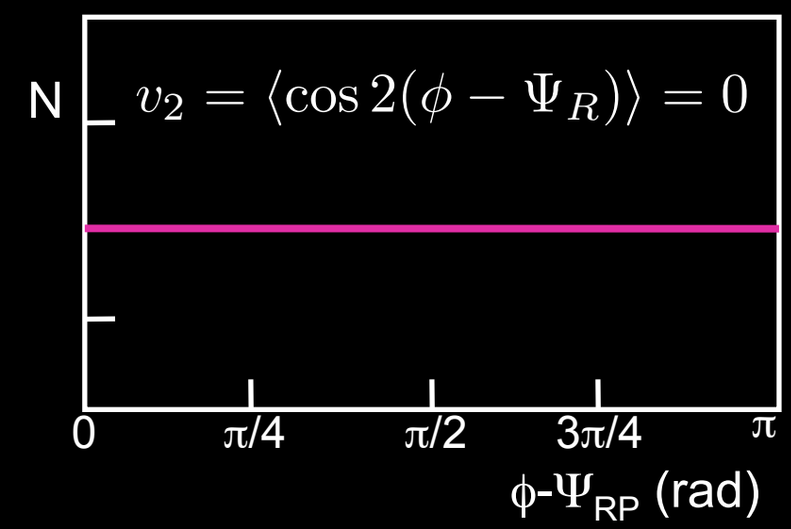

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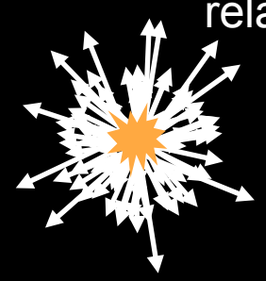

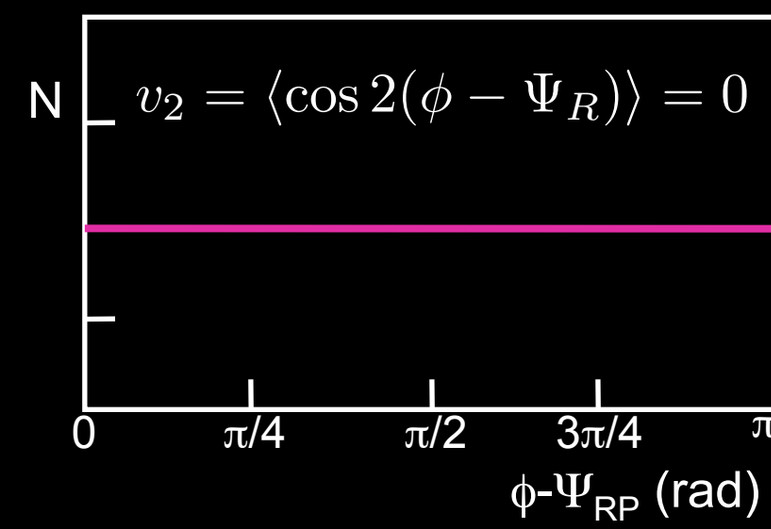

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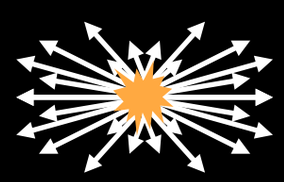
pressure gradients (larger in-plane)
push bulk "out" → "flow"



more, faster particles
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Elliptic Flow

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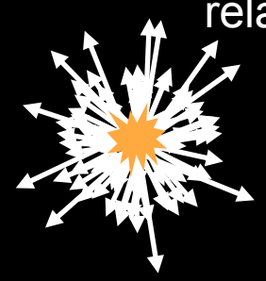

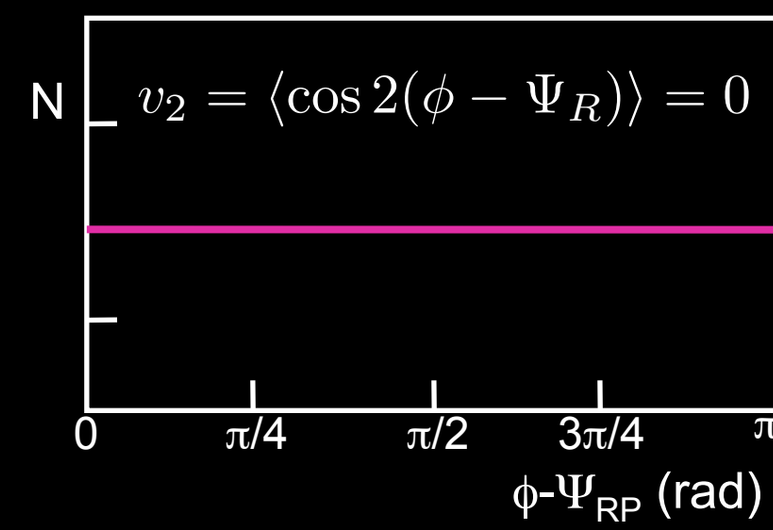

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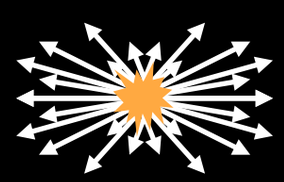

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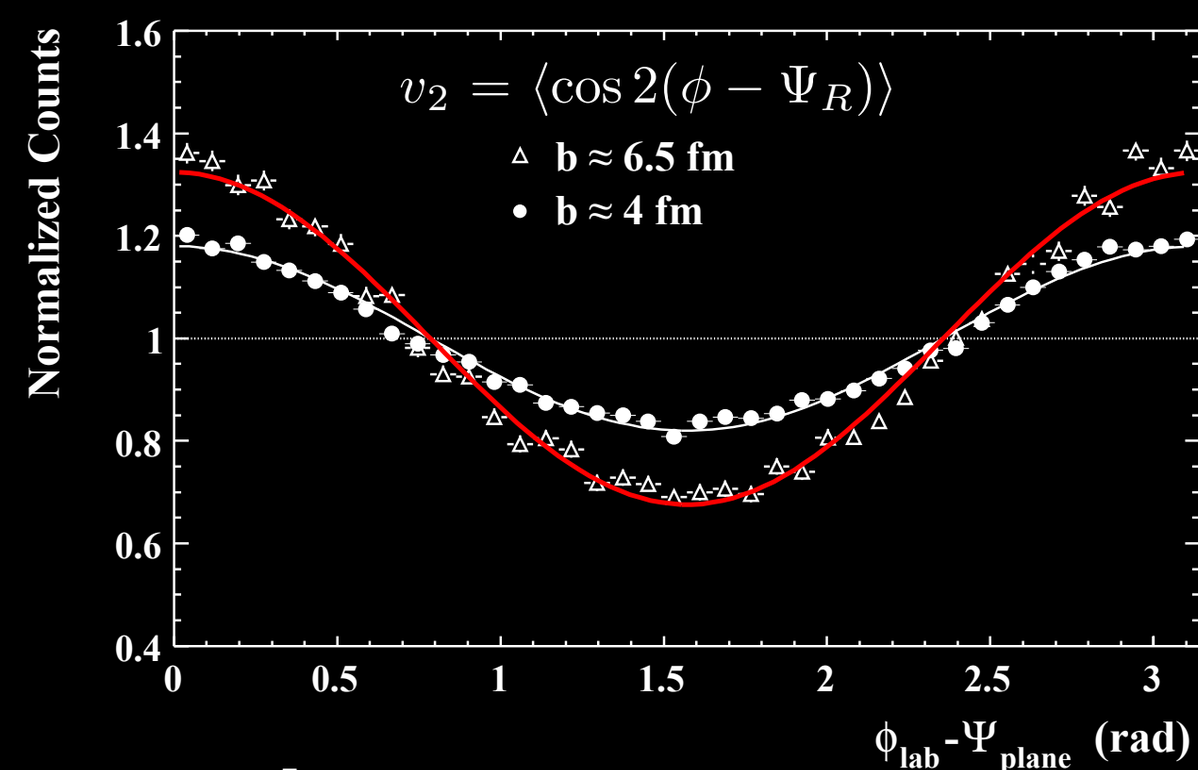
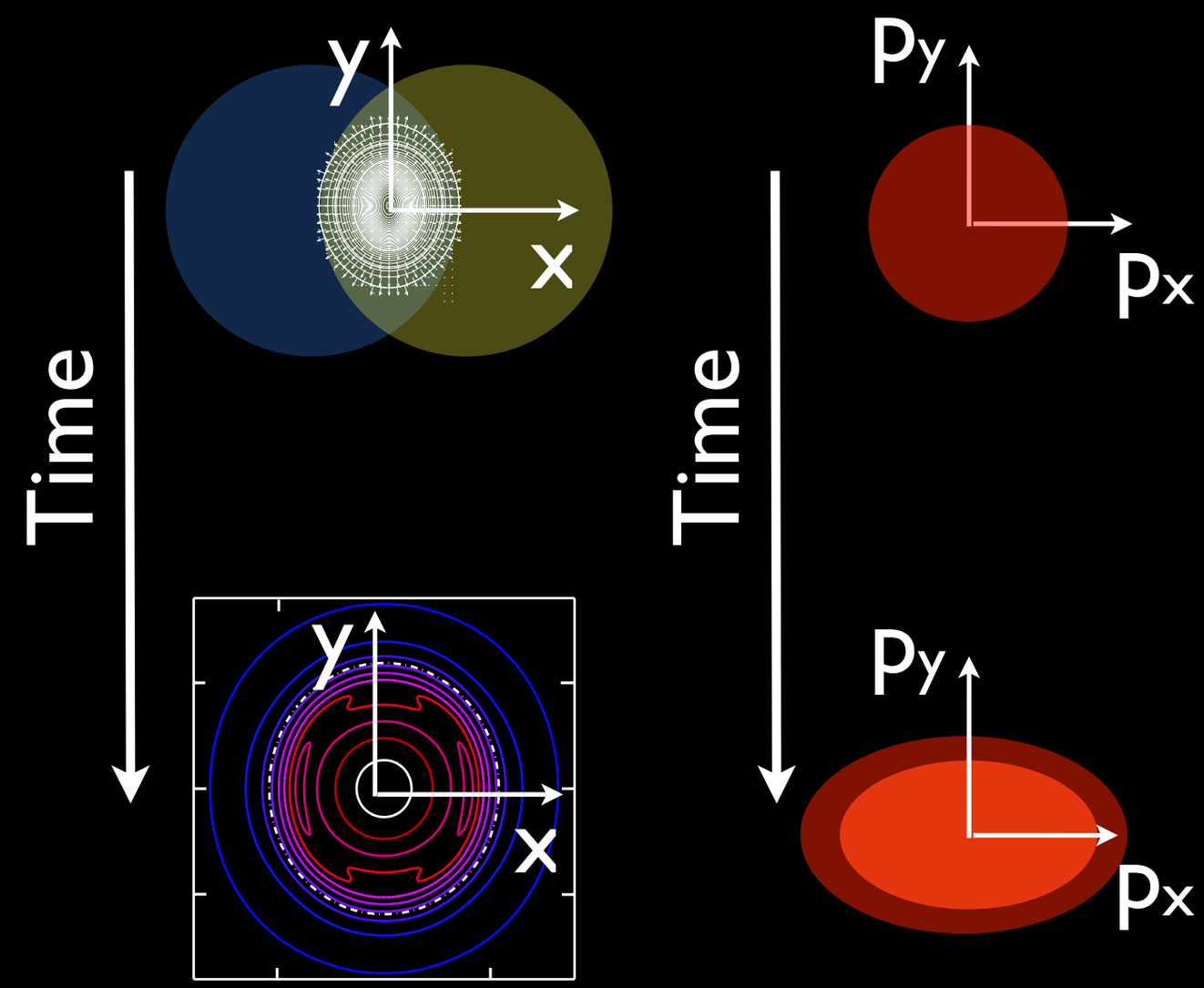


Elliptic Flow

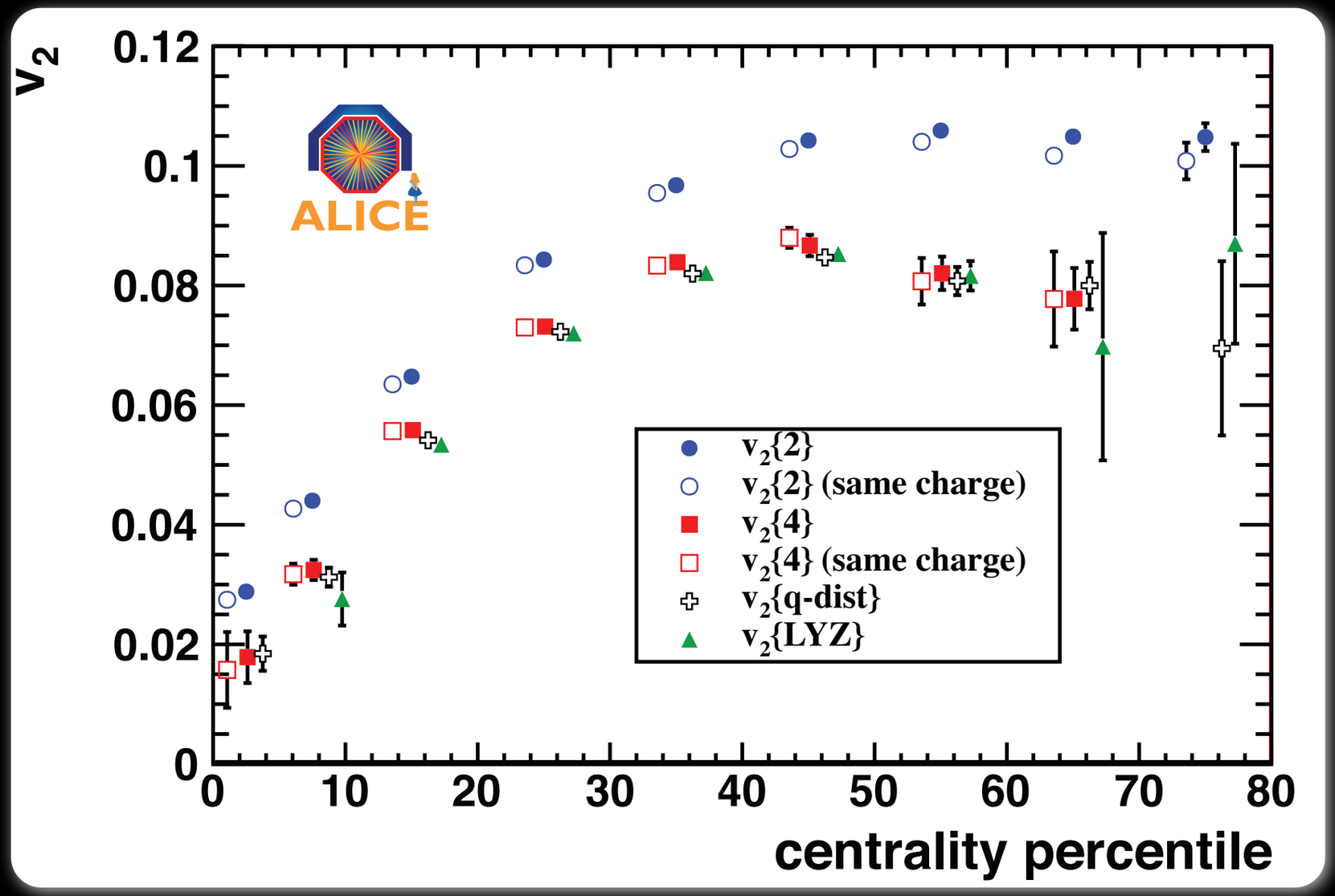
$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle} \qquad v_2 = \langle \cos 2\phi \rangle$$

- in non central collisions coordinate space configuration is anisotropic (almond shape). However, initial momentum distribution isotropic (spherically symmetric)
- interactions among constituents generate a pressure gradient which transforms the initial coordinate space anisotropy into the observed momentum space anisotropy → anisotropic flow
- self-quenching → sensitive to early stage



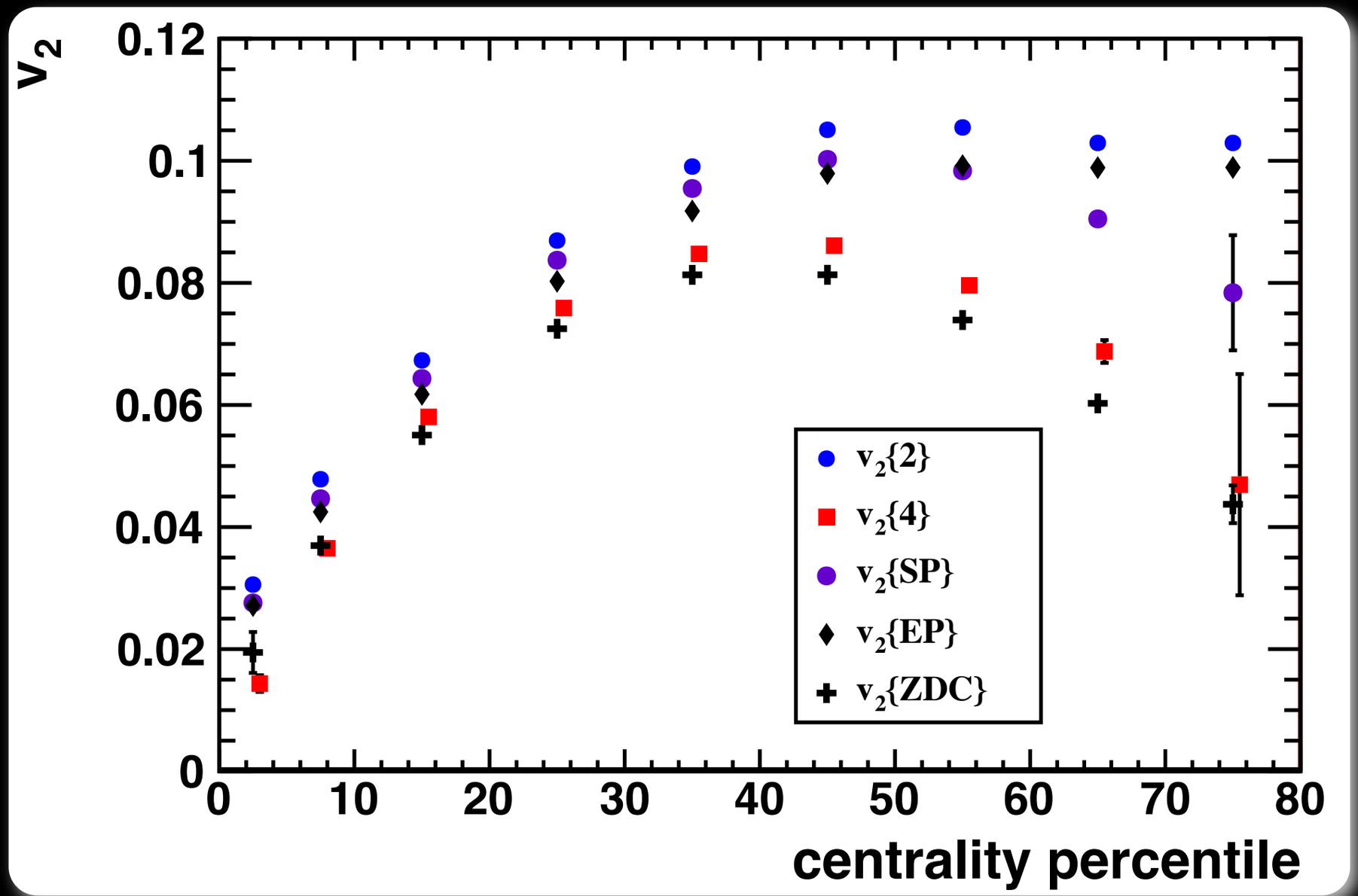
v_2 in ALICE

K. Aamodt et al. (ALICE Collaboration)
PRL 105, 252302 (2010)



Why are there two bands for the v_2 estimates?

charged particle v_2

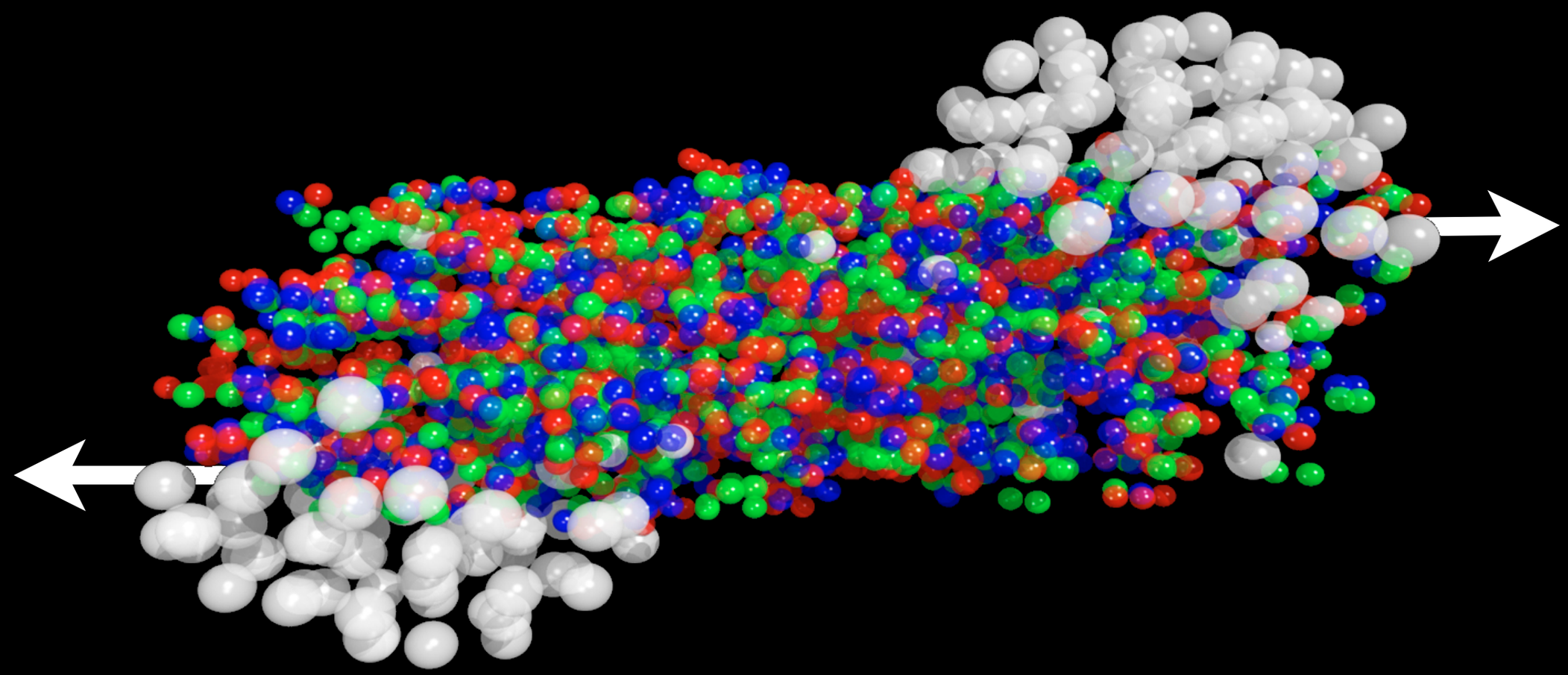


What causes these differences?

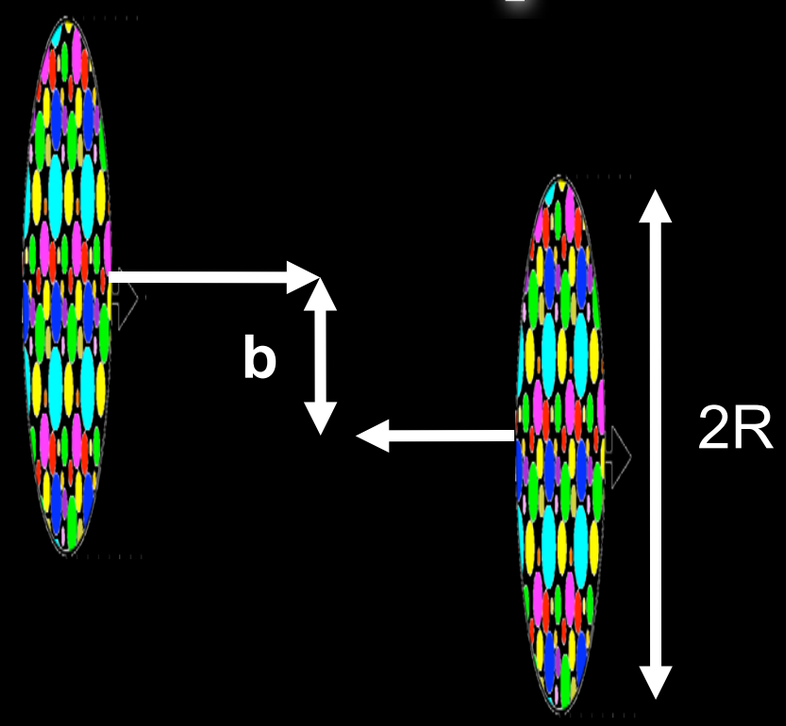
Content

Why are these different flow methods so different?

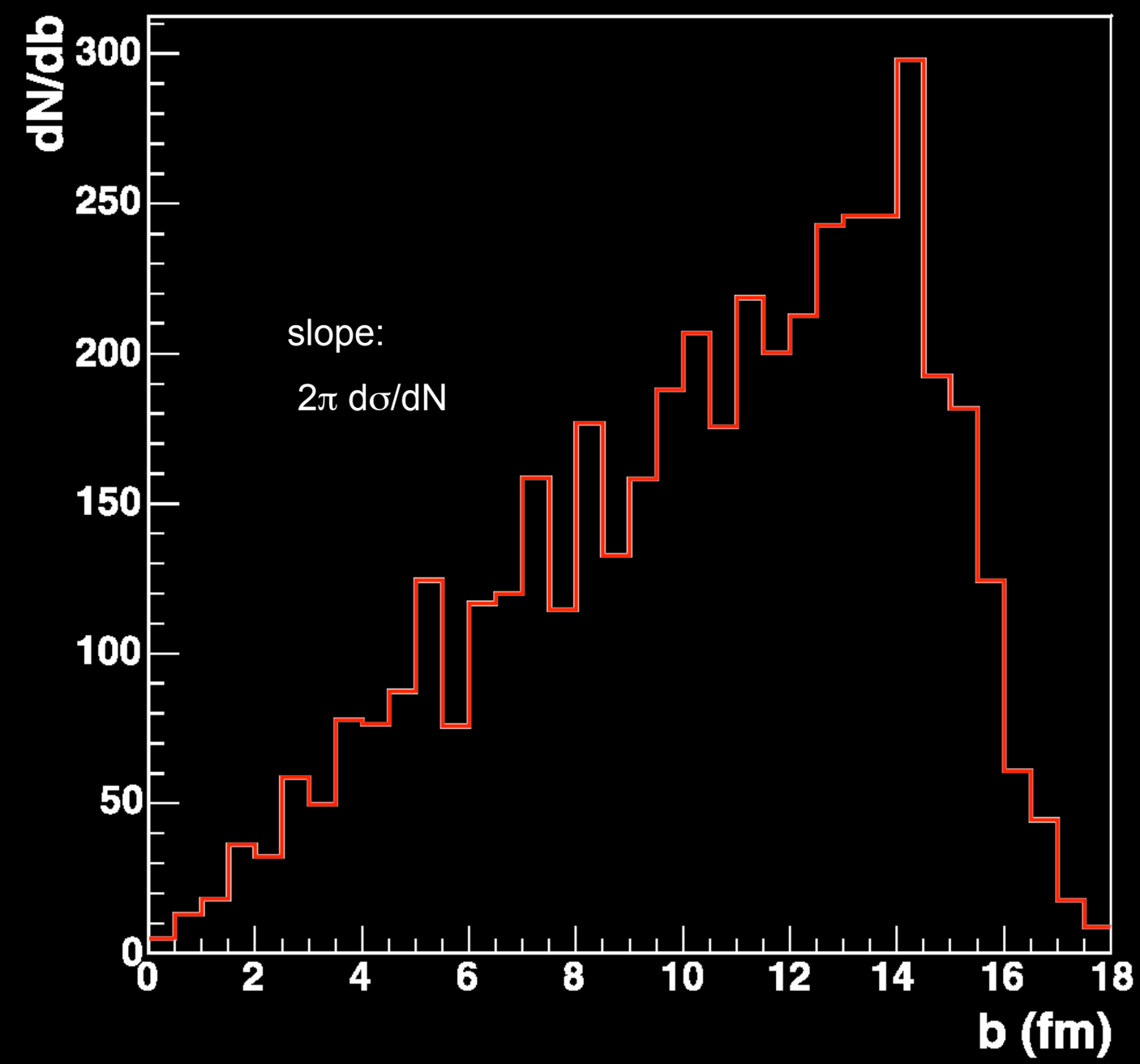
Event Characterization



Impact Parameter

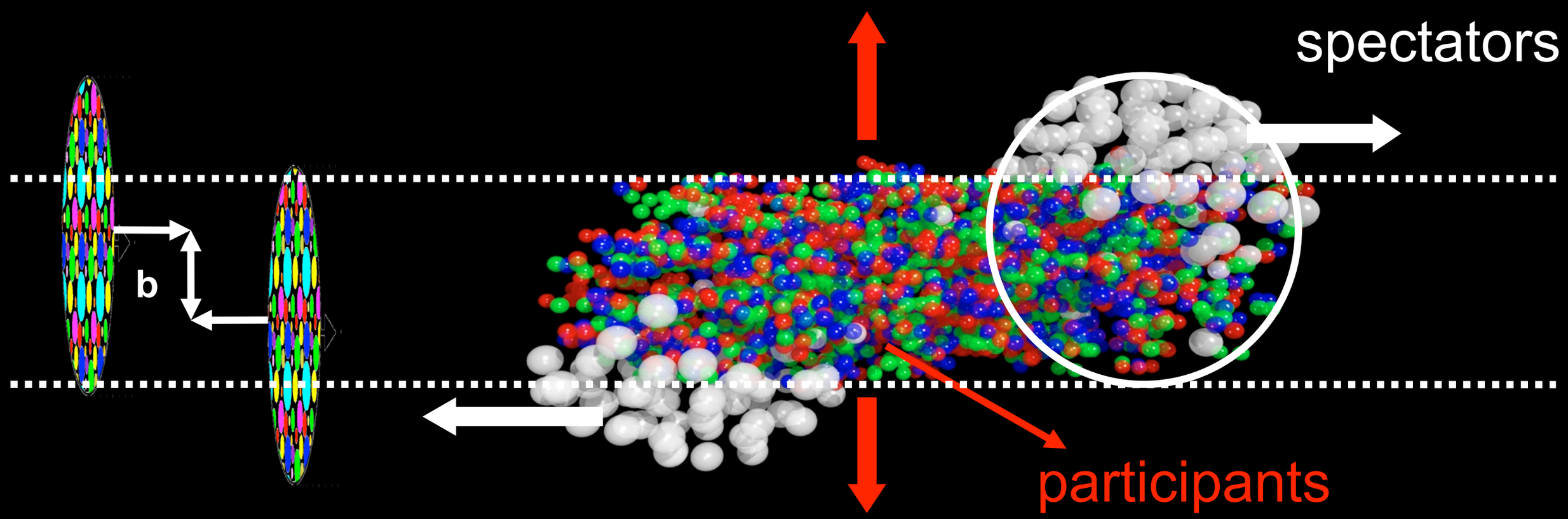


- impact parameter **b**
  - perpendicular to beam direction
  - connects centers of the colliding ions



$$d\sigma = 2\pi b db$$

Centrality Determination (I)



centrality characterized by:

1. N_{part}, $N_{wounded}$: number of nucleons which suffered at least one inelastic nucleon-nucleon collision
2. N_{coll}, N_{bin}: number of inelastic nucleon-nucleon collisions

Glauber Model Calculations

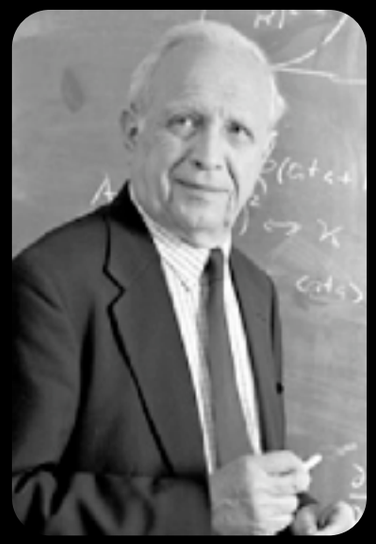
- ✓ nuclear density from Wood-Saxon distribution

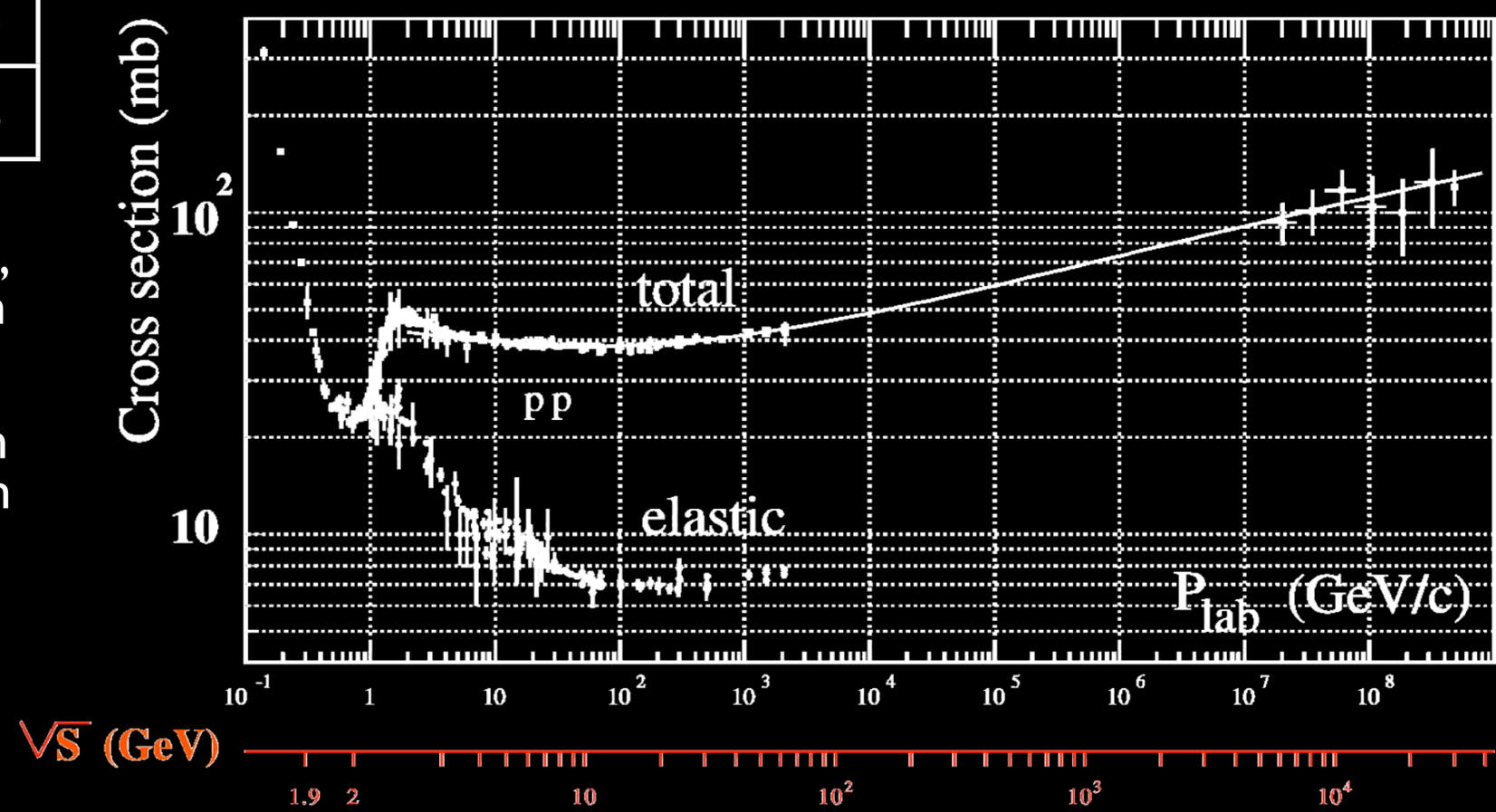
$$\rho(r) = \frac{\rho_0\left(1 + wr^2/R^2\right)}{1 + e^{(r-R)/a}}$$

Nucleus	A	R	a
Au	197	6.38	0.535
Pb	208	6.68	0.546

- ✓ nucleons travel on straight lines, no deflection after NN collision
- ✓ NN collision cross section from measured inelastic cross section in p+p
- ✓ NN cross section remains constant independent of how many collisions a nucleon suffered

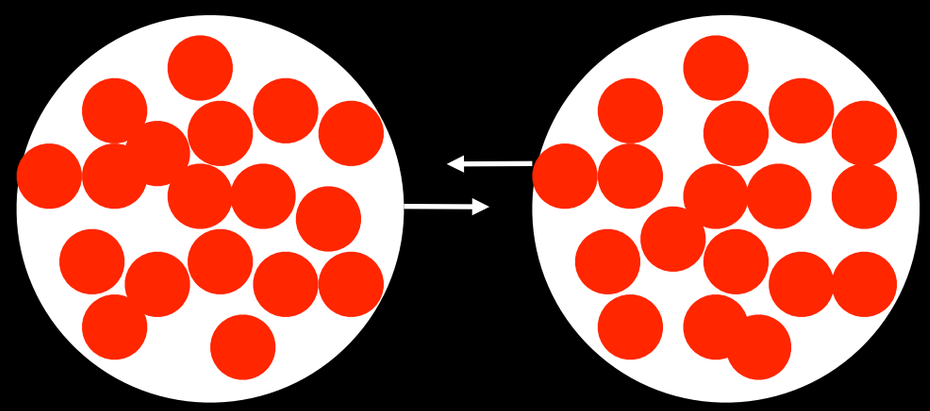
$\sqrt{S}$ (GeV)	$\sigma_{in,pp}$ (mb)
20	32
200	42
2700	~64





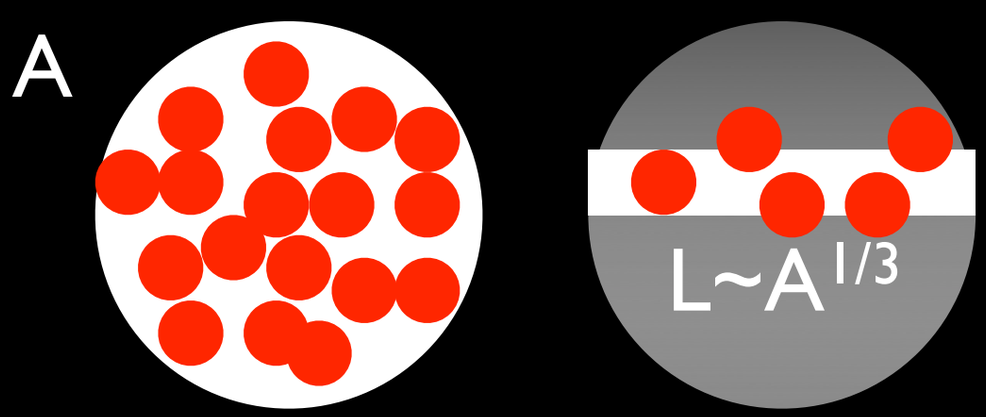
Wounded nucleons and binary collisions

wounded nucleon scaling

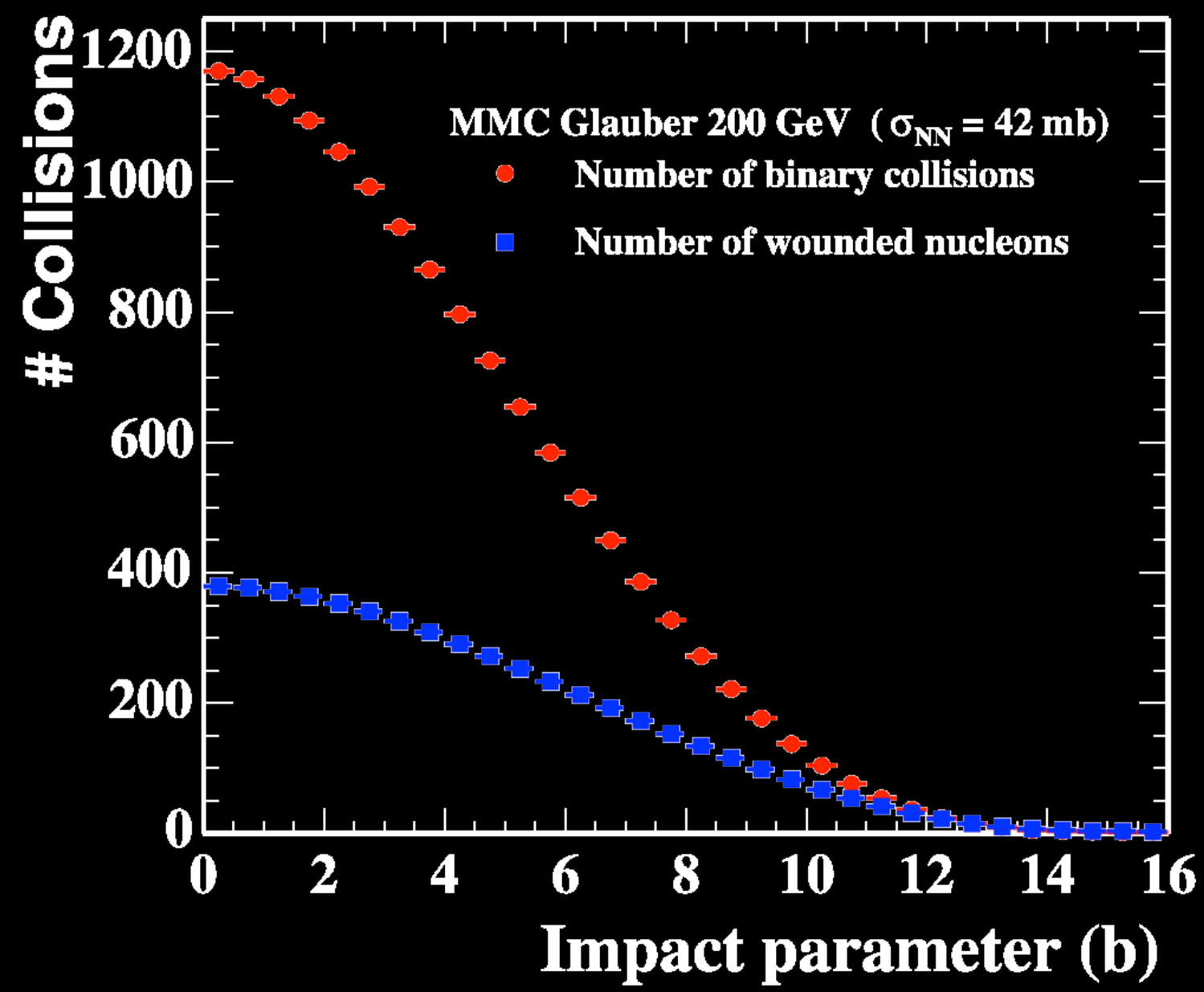


number of participating nucleons scales with volume ~ 2A

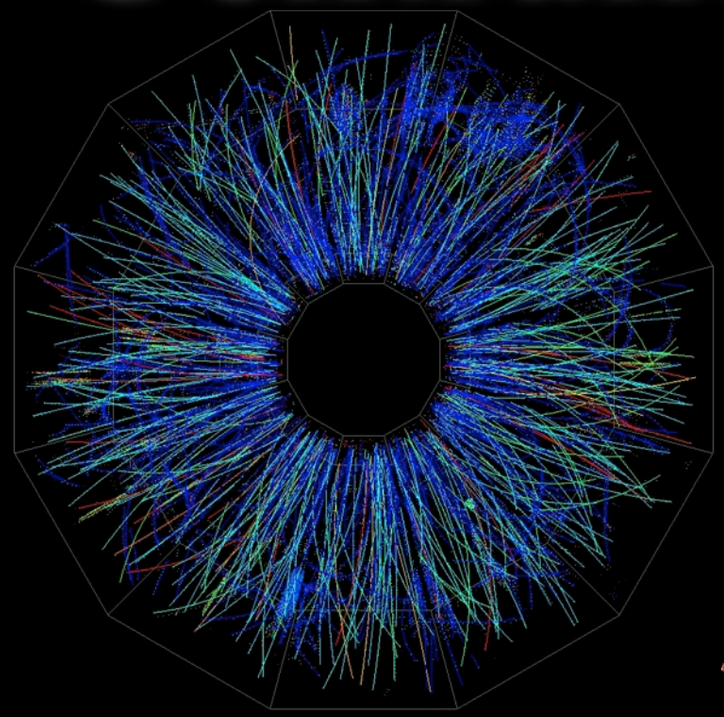
binary scaling

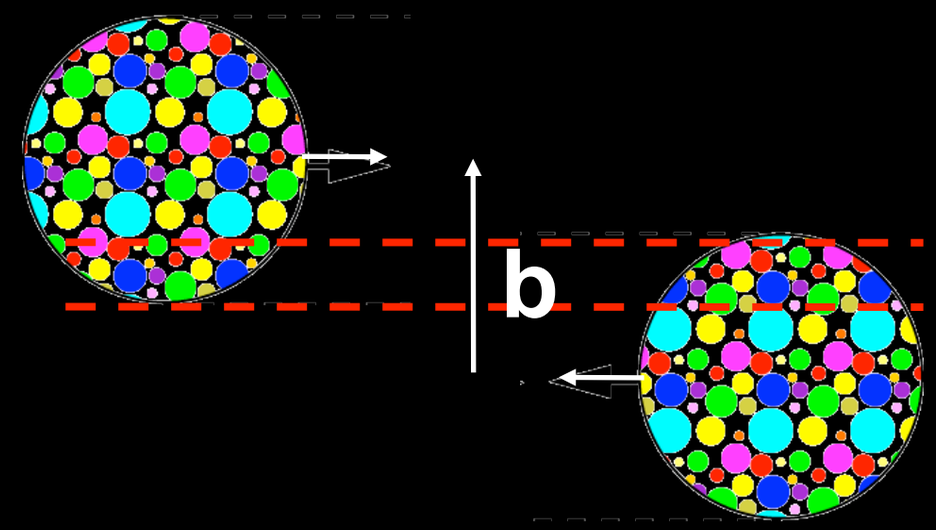


number of NN collisions, point like, scales with ~ $A^{4/3}$



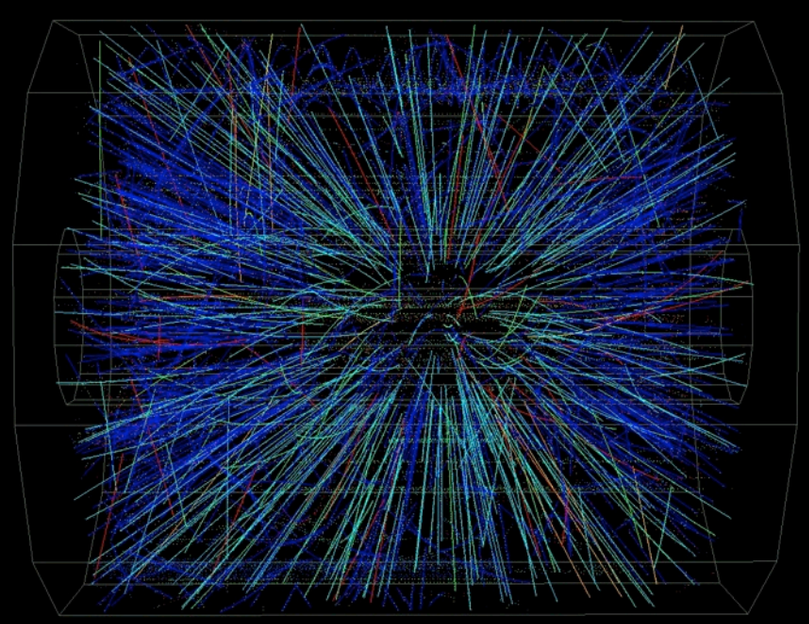
Centrality determination (III)





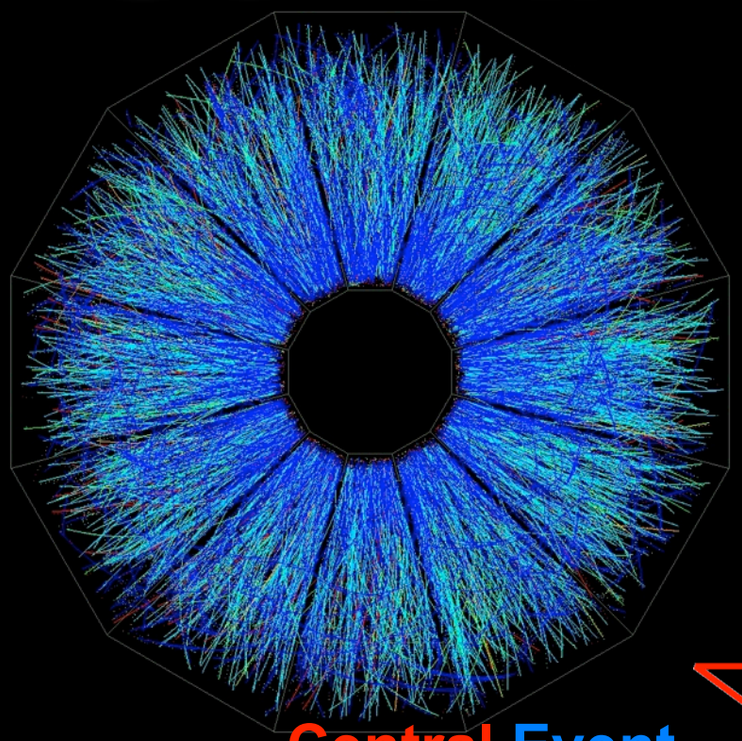
Peripheral Event

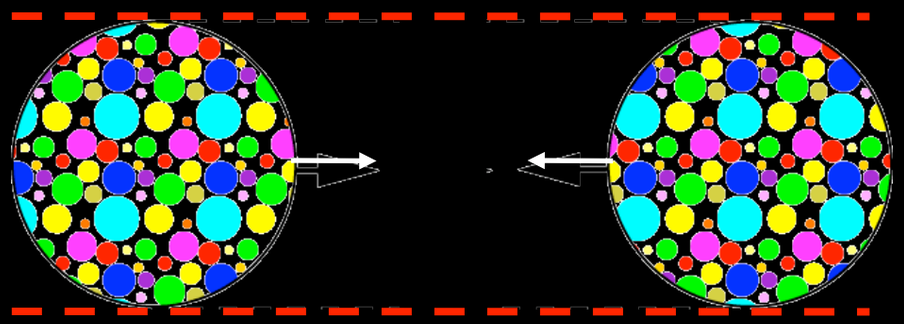
From real-time Level 3 display



- ✓ peripheral collisions, largest fraction cross section
- ✓ many spectators
- ✓ “few” particles produced

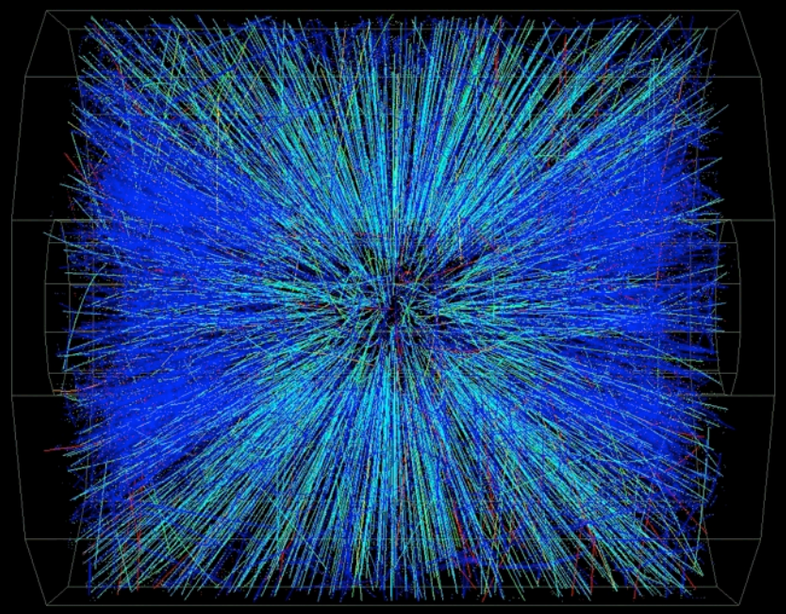
Centrality determination (IV)





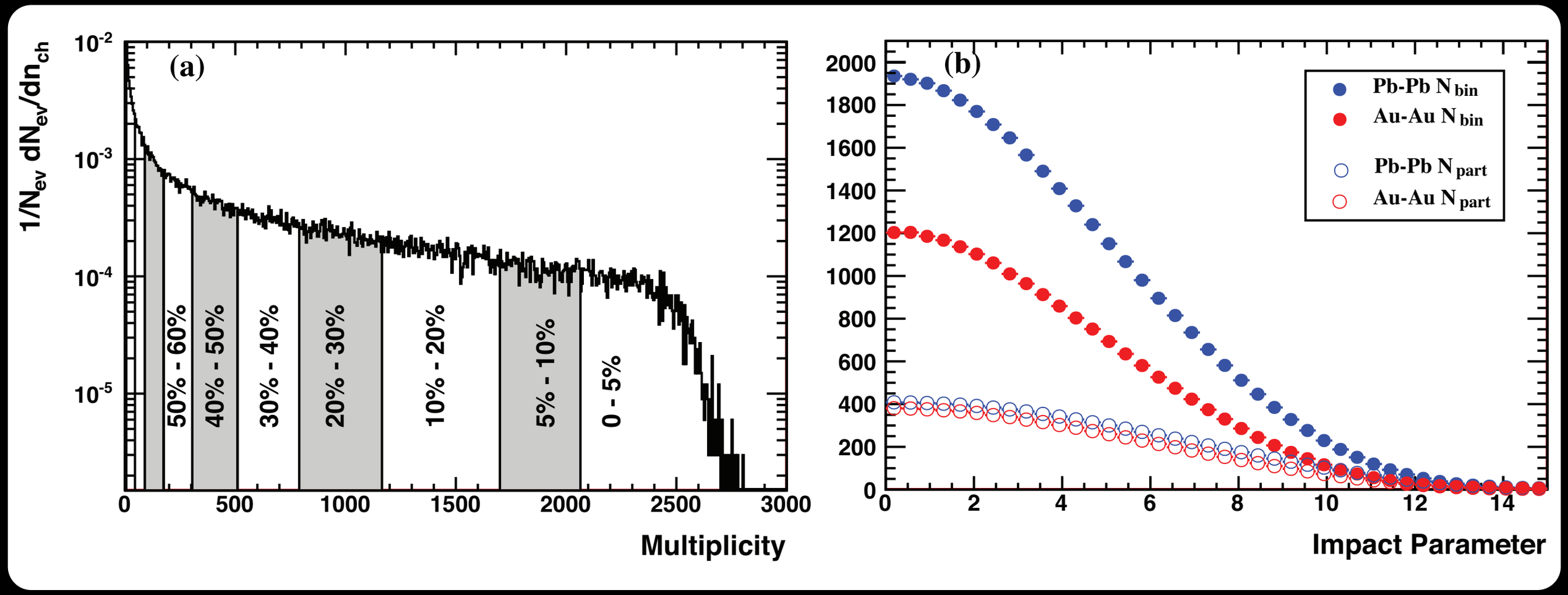
Central Event

From real-time Level 3 display



- ✓ impact parameter **b** = 0
- ✓ central collisions, small cross section
- ✓ no spectators
- ✓ many particles produced

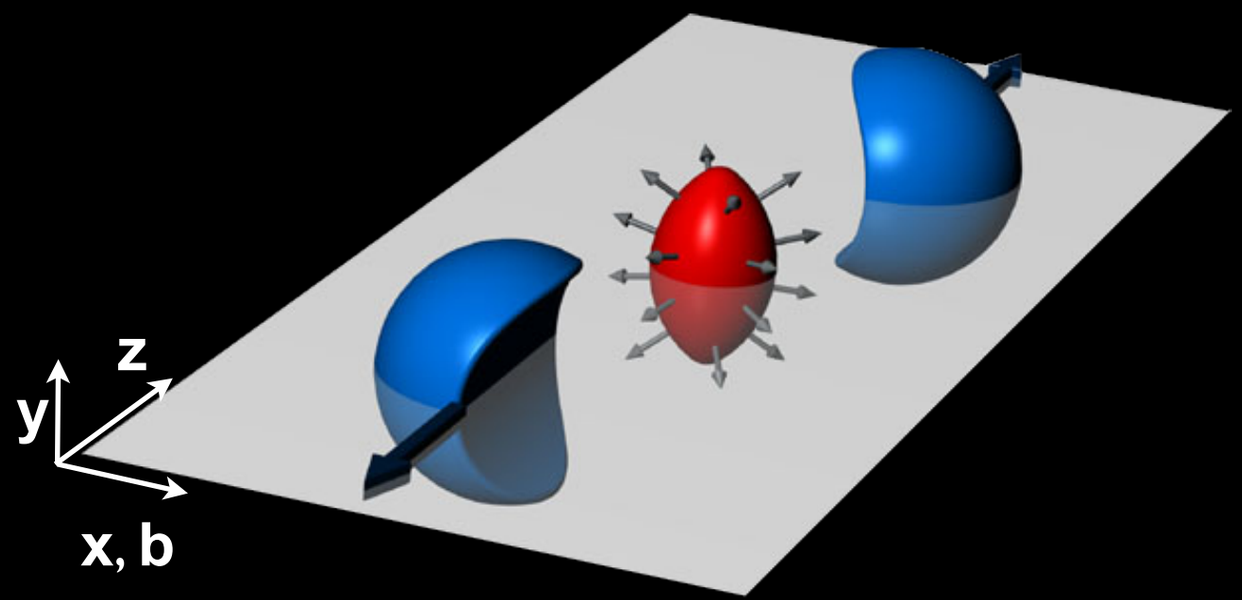
Centrality determination (ALICE)



✓ Determines the magnitude of the impact parameter

$\%\sigma_{tot}$	$\langle N_{part} \rangle$	$\langle b \rangle$
0-5	386	2.48
20-30	177	7.85
60-70	25	12.66

Anisotropic Flow



Azimuthal distributions of particles measured with respect to the reaction plane (spanned by impact parameter vector and beam axis) are not isotropic.

$$E\frac{d^3N}{d^3\vec{p}} = \frac{1}{2\pi}\frac{d^2N}{p_T\,dp_T\,dy}\left(1+\sum_{n=1}^{\infty}2v_n\cos\left(n\left(\phi-\Psi_{\mathrm{RP}}\right)\right)\right)$$

$$v_n = \langle \cos n(\phi - \Psi_{\mathrm{RP}})\rangle$$

harmonics v_n quantify anisotropic flow

S. Voloshin and Y. Zhang (1996)

measure anisotropic flow

- since reaction plane cannot be measured event-by-event, consider quantities which do not depend on it's orientation: multi-particle azimuthal correlations

$$\left\langle e^{in(\phi_1-\phi_2)} \right\rangle = \left\langle e^{in\phi_1} \right\rangle \left\langle e^{-in\phi_2} \right\rangle + \left\langle e^{in(\phi_1-\phi_2)} \right\rangle_{\mathrm{corr}}$$

zero for symmetric detector when averaged over many events

$$\begin{aligned} \langle\langle 2 \rangle\rangle \equiv \left\langle\left\langle e^{in(\phi_1-\phi_2)} \right\rangle\right\rangle &= \left\langle\left\langle e^{in(\phi_1-\Psi_{\mathrm{RP}}-(\phi_2-\Psi_{\mathrm{RP}}))} \right\rangle\right\rangle \\ &= \left\langle\left\langle e^{in(\phi_1-\Psi_{\mathrm{RP}})} \right\rangle \left\langle e^{-in(\phi_2-\Psi_{\mathrm{RP}})} \right\rangle\right\rangle \\ &= \left\langle v_n^2 \right\rangle \end{aligned}$$

- assuming that <u>only</u> correlations with the reaction plane are present

intermezzo

- why do we define the correlations like this:

$$\left\langle \langle x \rangle_{\text{particles in single event}} \right\rangle_{\text{over events}}$$

  - easy to relate to v_n

$$\left\langle\left\langle e^{in(\phi_1-\phi_2)} \right\rangle\right\rangle$$

  - vanishes for independent particles

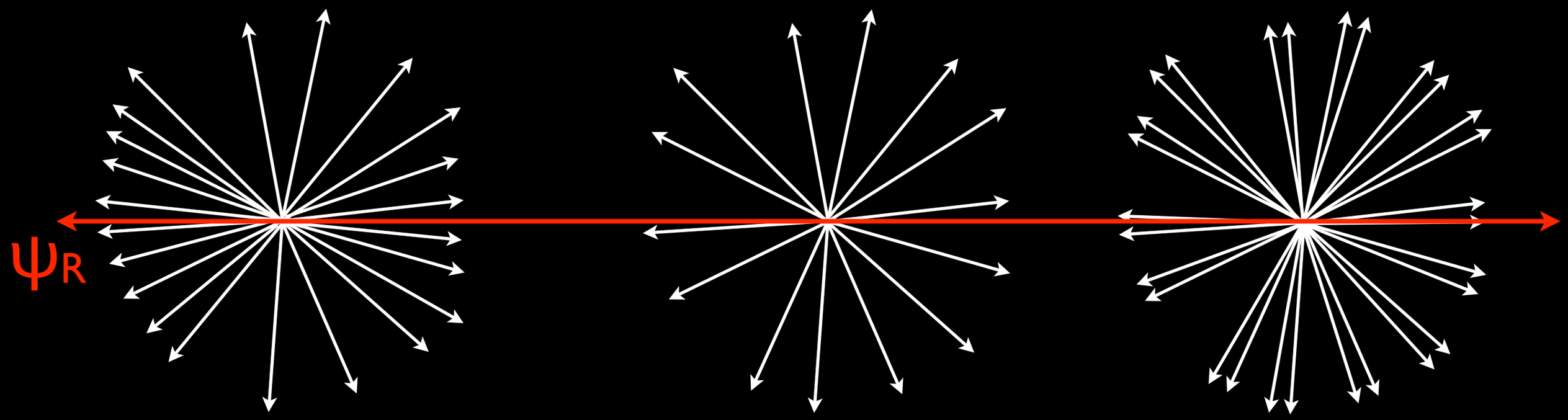
$$\left\langle\left\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \right\rangle\right\rangle$$

  - do not depend on frame $\Phi + \alpha$ (shifting all particles by fixed angle) gives same answer for the correlation

nonflow

- however, there are other sources of correlations between the particles which are not related to the reaction plane which break the factorization, lets call those δ_2 for two particle correlations

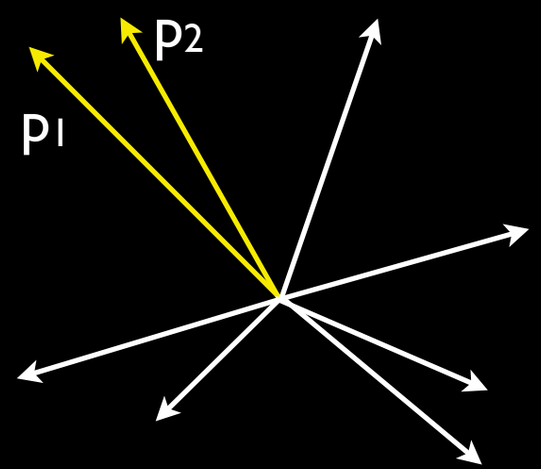
$$\left\langle\left\langle e^{in(\phi_1-\phi_2)}\right\rangle\right\rangle = \langle v_n^2\rangle + \delta_2$$



$v_2 > 0, v_2\{2\} > 0$ $\quad$ $v_2 = 0, v_2\{2\} = 0$ $\quad$ $v_2 = 0, v_2\{2\} > 0$

nonflow

$$\left\langle\left\langle e^{in(\phi_1-\phi_2)}\right\rangle\right\rangle = \langle v_n^2\rangle + \delta_2$$



particle 1 coming from the resonance. Out of remaining M-1 particles there is only one which is coming from the same resonance, particle 2. Hence a probability that out of M particles we will select two coming from the same resonance is ~ 1/(M-1). From this we can draw a conclusion that for large multiplicity: $\delta_2 \sim 1/M$

- therefore to reliably measure flow:

$$v_n^2 \gg 1/M \quad \Rightarrow \quad v_n \gg 1/M^{1/2}$$

- not easily satisfied: M=200 v_n >> 0.07

can we do better?

- use the fact that flow is a correlation between all particles: use multi-particle correlations

$$\left\langle\left\langle e^{in(\phi_1-\phi_2)}\right\rangle\right\rangle = v_n^2 + \delta_2$$

$$\left\langle\left\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)}\right\rangle\right\rangle = v_n^4 + 4v_n^2\delta_2 + 2\delta_2^2 + \delta_4$$

- not so clear if we gained something

Can we do better?

- build cumulants with the multi-particle correlations

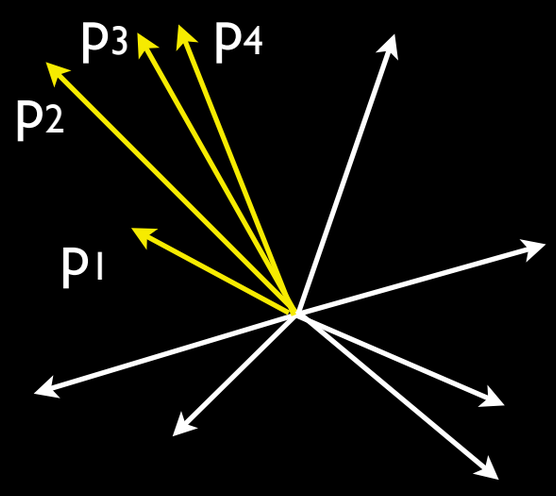
Ollitrault and Borghini

- for detectors with uniform acceptance 2nd and 4th cumulant are given by:

$$
\begin{aligned}
c_n\{2\} &\equiv \left\langle\!\!\left\langle e^{in(\phi_1-\phi_2)} \right\rangle\!\!\right\rangle = v_n^2 + \delta_2 \\
c_n\{4\} &\equiv \left\langle\!\!\left\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \right\rangle\!\!\right\rangle - 2\left\langle\!\!\left\langle e^{in(\phi_1-\phi_2)} \right\rangle\!\!\right\rangle^2 \\
&= v_n^4 + 4v_n^2\delta_2 + 2\delta_2^2 - 2(v_n^2+\delta_2)^2 + \delta_4 \\
&= -v_n^4 + \delta_4
\end{aligned}
$$

- got rid of two particle non-flow correlations!

Can we do better?



Particle 1 coming from the mini-jet. To select particle 2 we can make a choice out of remaining M-1 particles; once particle 2 is selected we can select particle 3 out of remaining M-2 particles and finally we can select particle 4 out of remaining M-3 particles. Hence the probability that we will select randomly four particles coming from the same resonance is 1/(M-1)(M-2)(M-3). From this we can draw a conclusion that for large multiplicity:

$$\delta_2 \sim 1/M, \qquad \delta_4 \sim 1/M^3$$

- therefore to reliably measure flow:

$$v_n^2 \gg 1/M \quad \Rightarrow \quad v_n \gg 1/M^{1/2}$$
$$v_n^4 \gg 1/M^3 \quad \Rightarrow \quad v_n \gg 1/M^{3/4}$$

Can we do better?

- it is possible to extend this:

$$v_n^{2k} \gg 1/M^{2k-1} \Rightarrow v_n \gg 1/M^{\frac{2k-1}{2k}}$$

- for large k (or even M particle correlations e.g. Lee Yang Zeroes)

$$v_n \gg 1/M$$

- as an example: M=200 v_n >> 0.005 (more than order of magnitude better than two particle correlations)
- to reliably measure small flow in presence of other correlations one needs to use multi-particle correlations!

Calculate Correlations

(using nested loops)

To evaluate average 2-particle correlation

$$\langle 2 \rangle \equiv \left\langle e^{in(\phi_1 - \phi_2)} \right\rangle = \frac{1}{\binom{M}{2} 2!} \sum_{\substack{i,j=1 \\ (i \neq j)}}^{M} e^{in(\phi_i - \phi_j)}$$

in a nested loop # operations $\frac{1}{2!} \frac{M!}{(M-2)!}$

- With M=1000, this approach already for 4-particle correlations gives 1.2×10^{12} operations per event!
- calculation of average 6-particle correlation requires roughly 1.4×10^{17} operations, and of average 8-particle correlation roughly 8.4×10^{21} operations per event
- clearly not the way to go

Calculate Correlations

(using Q-cumulants)

A. Bilandzic, RS, S. Voloshin (2011)

azimuthal two particle correlations:

$$\langle 2 \rangle \equiv \left\langle e^{in(\phi_1 - \phi_2)} \right\rangle = \frac{1}{\binom{M}{2} 2!} \sum_{\substack{i,j=1 \\ (i \neq j)}}^{M} e^{in(\phi_i - \phi_j)}$$

definition of Q vector of harmonic n

$$Q_n \equiv \sum_{i=1}^{M} e^{in\phi_i}$$

can write two particle correlation in terms of Q vector of harmonic n

$$\langle 2 \rangle = \frac{|Q_n|^2 - M}{M(M-1)}$$

Calculate Correlations

(using Q-cumulants)

as we saw before in case of only flow correlations

$$\langle 2 \rangle = \frac{|Q_n|^2 - M}{M(M-1)} = v_n^2$$

for zero flow, we have a random walk

$$|Q_n| \sim \sqrt{M} \qquad \langle 2 \rangle = \frac{M - M}{M(M-1)} = 0$$

and as we saw if there is no flow and only nonflow we get

$$\langle 2 \rangle = \delta_2$$

Calculate Correlations

(using Q-cumulants)

two particle correlations can be expressed in Q vectors

$$\langle 2 \rangle = \frac{|Q_n|^2 - M}{M(M-1)}$$

but also four particle correlations (and more)

note the mixed harmonics

$$\langle 4 \rangle = \frac{|Q_n|^4 + |Q_{2n}|^2 - 2 \cdot \mathbf{Re}\left[Q_{2n} Q_n^* Q_n^*\right] - 4(M-2) \cdot |Q_n|^2}{M(M-1)(M-2)(M-3)} + \frac{2}{(M-1)(M-2)}$$

with this it becomes trivial to make cumulants again

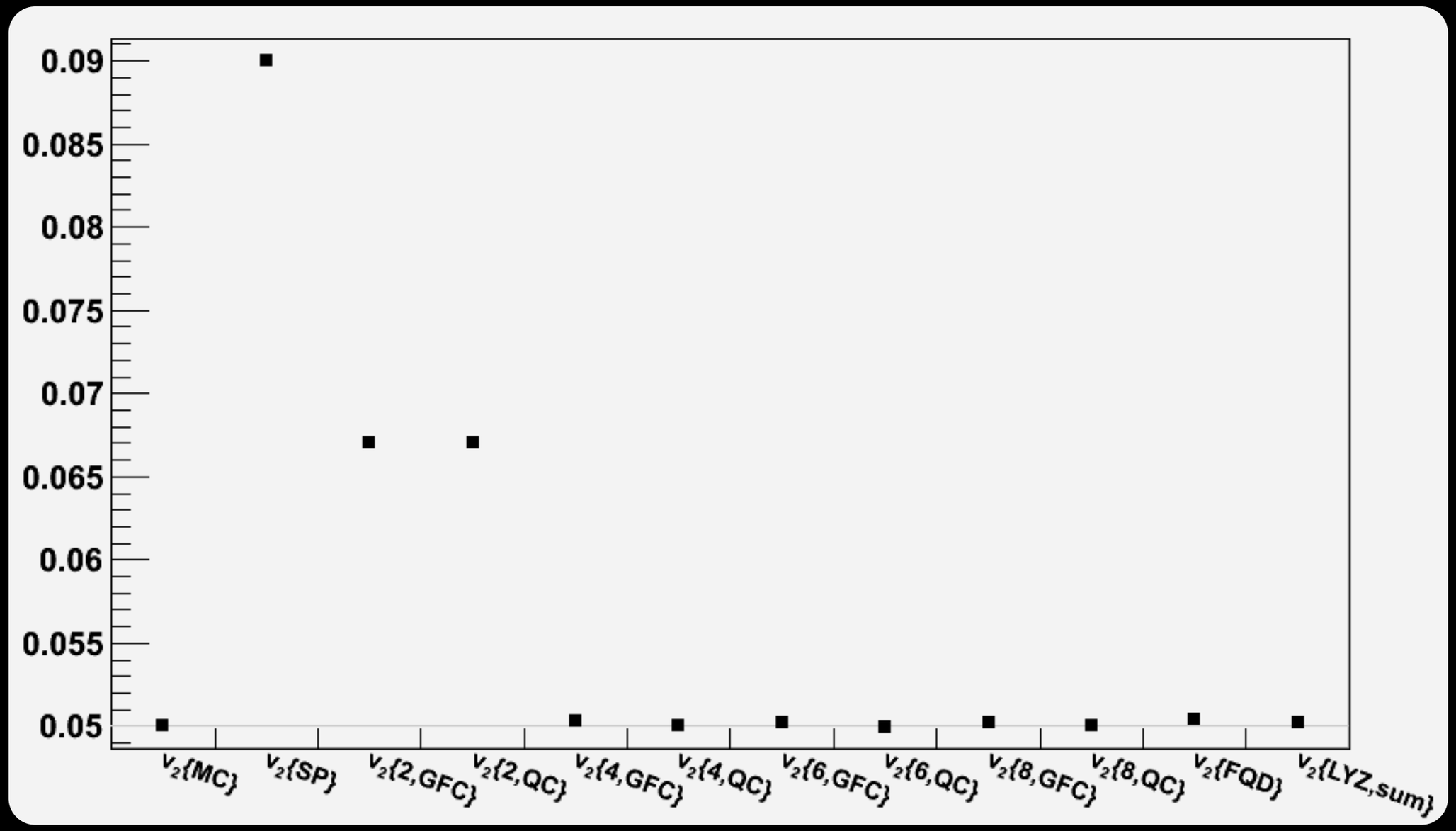
Calculate Correlations

(using Q-cumulants)

- pros Q-cumulants
  - exact solutions, give same answer as nested loops
  - one loop over data enough to calculate all multi-particle correlations
  - number of operations to get all multi-particle correlations up to 8th order is 4 x 2 x Multiplicity
    - for multiplicities of ~ 1000 the number of operations is reduced by a factor 10^{18} !!

nonflow example

Example: input $v_2 = 0.05$, $M = 500$, $N = 5 \times 10^6$ and simulate nonflow by taking each particle twice



as expected only two particle methods are biased

Flow Fluctuations

Both two and multi-particle correlations have an extra feature one has to keep in mind!

- By using multi-particle correlations to estimate flow we are actually estimating the averages of various powers of flow

$$\langle\langle 2\rangle\rangle = \left\langle v^2 \right\rangle, \quad \langle\langle 6\rangle\rangle = \left\langle v^6 \right\rangle$$
$$\langle\langle 4\rangle\rangle = \left\langle v^4 \right\rangle, \quad \langle\langle 8\rangle\rangle = \left\langle v^8 \right\rangle$$

- But what we are after is: $\langle v \rangle$

Flow Fluctuations

- in general: take a random variable x with mean μ_x and spread σ_x . The the expectation value of some function of a random variable x, E[h(x)], is to leading order given by

$$\langle h(x) \rangle \equiv E[h(x)] = h(\mu_x) + \frac{\sigma_x^2}{2} h''(\mu_x)$$

- using this for the flow results:

$$\begin{aligned}
\langle v^2 \rangle &= \langle v \rangle^2 + \sigma_v^2 \\
\langle v^4 \rangle &= \langle v \rangle^4 + 6\sigma_v^2 \langle v \rangle^2 \\
\langle v^6 \rangle &= \langle v \rangle^6 + 15\sigma_v^2 \langle v \rangle^4 \\
\langle v^8 \rangle &= \langle v \rangle^8 + 28\sigma_v^2 \langle v \rangle^6
\end{aligned}$$

- remember cumulants are combinations of these quantities

Flow Fluctuations

- flow estimates from cumulants can be written as:

$$
\begin{aligned}
v\{2\} &= \left\langle v^2 \right\rangle^{1/2} \\
v\{4\} &= \left(-\left\langle v^4 \right\rangle + 2\left\langle v^2 \right\rangle^2\right)^{1/4} \\
v\{6\} &= \left[\frac{1}{4}\left(\left\langle v^6 \right\rangle - 9\left\langle v^2 \right\rangle\left\langle v^4 \right\rangle + 12\left\langle v^2 \right\rangle^3\right)\right]^{1/6} \\
v\{8\} &= \left[-\frac{1}{33}\left[\left\langle v^8 \right\rangle - 16\left\langle v^6 \right\rangle\left\langle v^2 \right\rangle - 18\left\langle v^4 \right\rangle^2\right.\right. \\
&\qquad \left.\left. +144\left\langle v^4 \right\rangle\left\langle v^2 \right\rangle^2 - 144\left\langle v^2 \right\rangle^4\right]\right]^{1/8}
\end{aligned}
$$

- take the expression from previous slide and use:

$$\sigma_v \ll \langle v \rangle$$

- take up to order σ^2, the surprisingly simple result is:

Flow Fluctuations

$$v\{2\} = \langle v \rangle + \frac{1}{2}\frac{\sigma_v^2}{\langle v \rangle}$$

$$v\{4\} = \langle v \rangle - \frac{1}{2}\frac{\sigma_v^2}{\langle v \rangle}$$

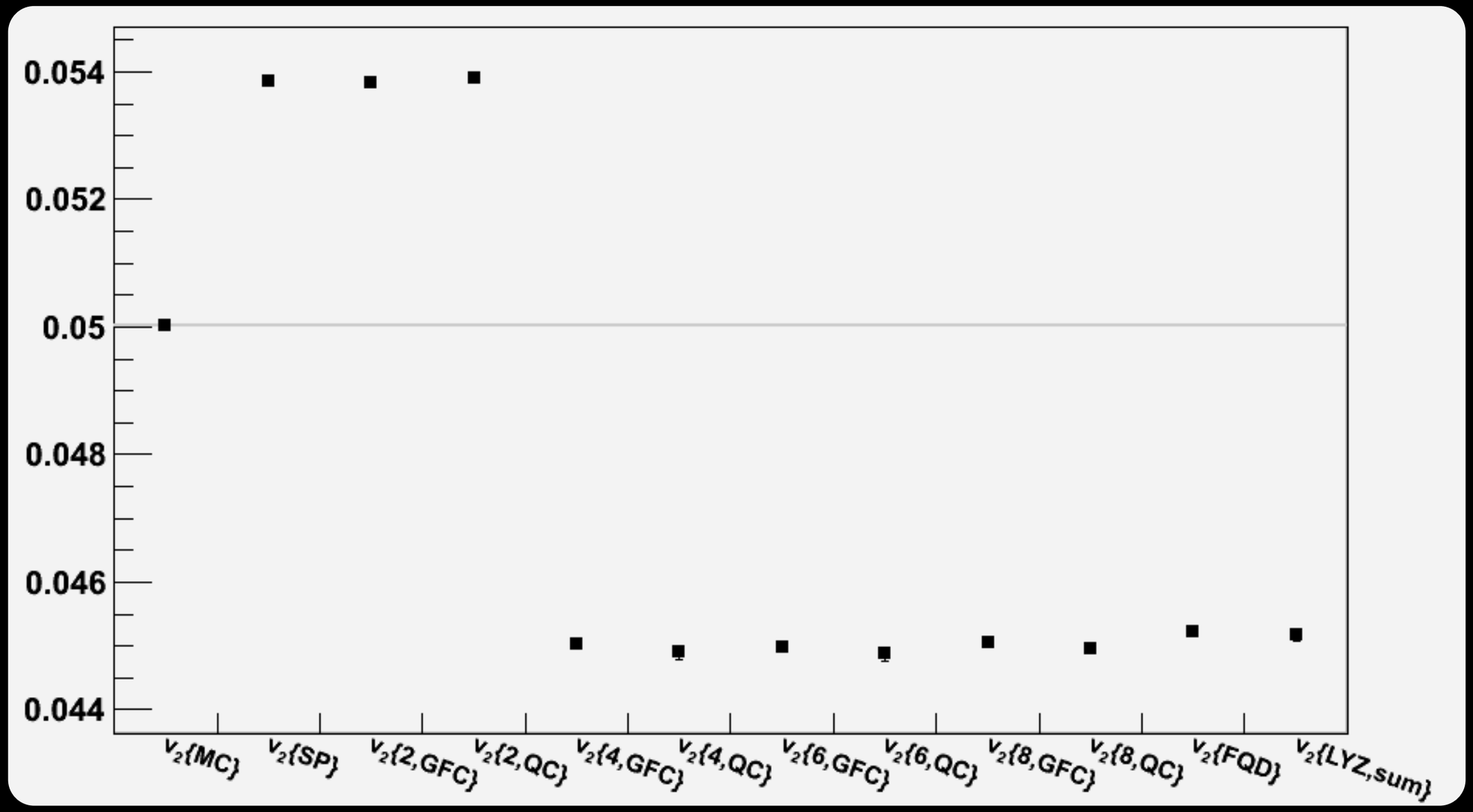
$$v\{6\} = \langle v \rangle - \frac{1}{2}\frac{\sigma_v^2}{\langle v \rangle}$$

$$v\{8\} = \langle v \rangle - \frac{1}{2}\frac{\sigma_v^2}{\langle v \rangle}$$

- for $\sigma_v \ll \langle v \rangle$ this is a general result to order σ^2

Flow Fluctuations

Example: input v_2 = 0.05 +/- 0.02 (Gausian), M = 500, N = 1 × 10^6



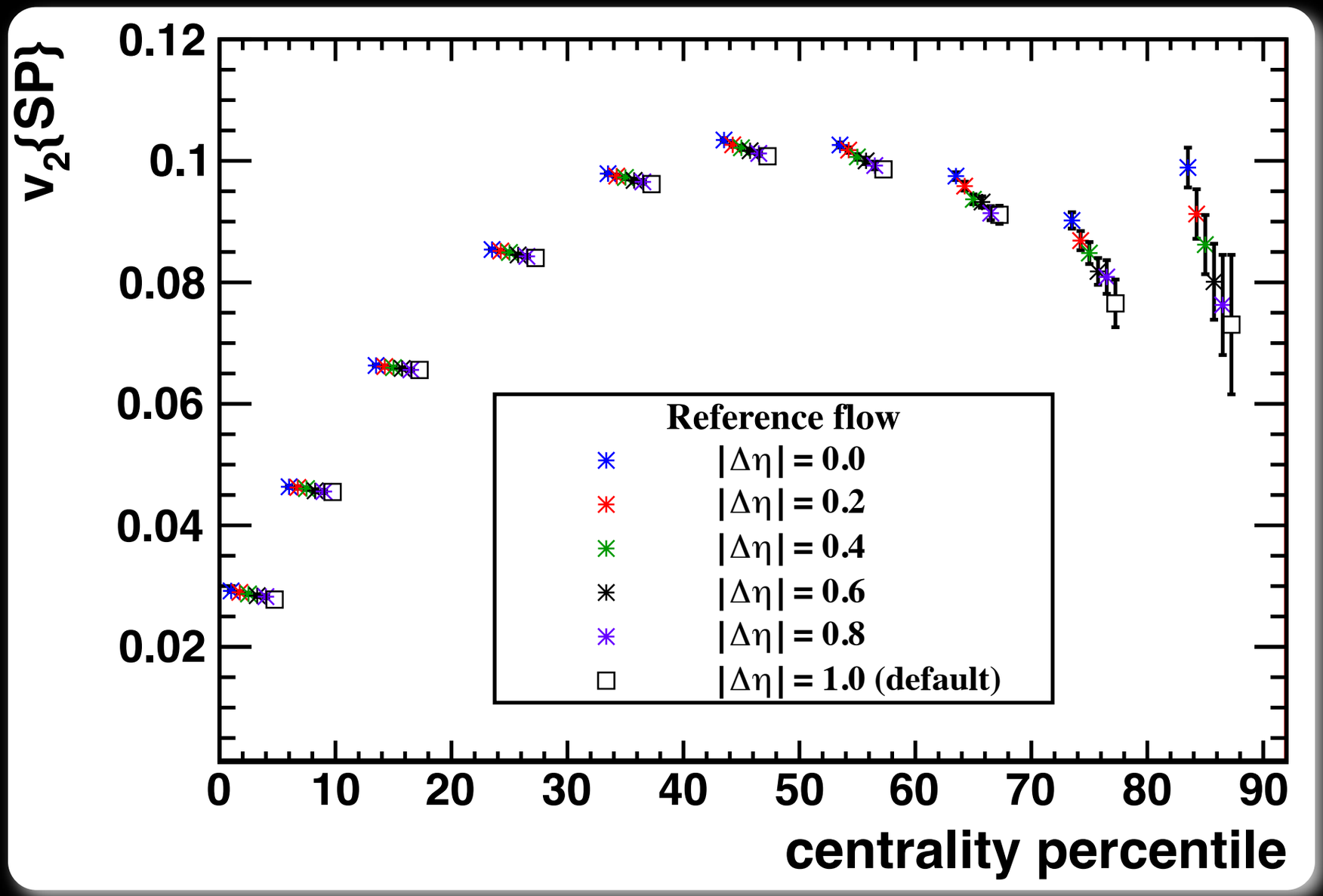
Gaussian fluctuation behave as predicted also for Lee Yang Zeroes and fitting Q distribution (more on that later)

Summary Methods

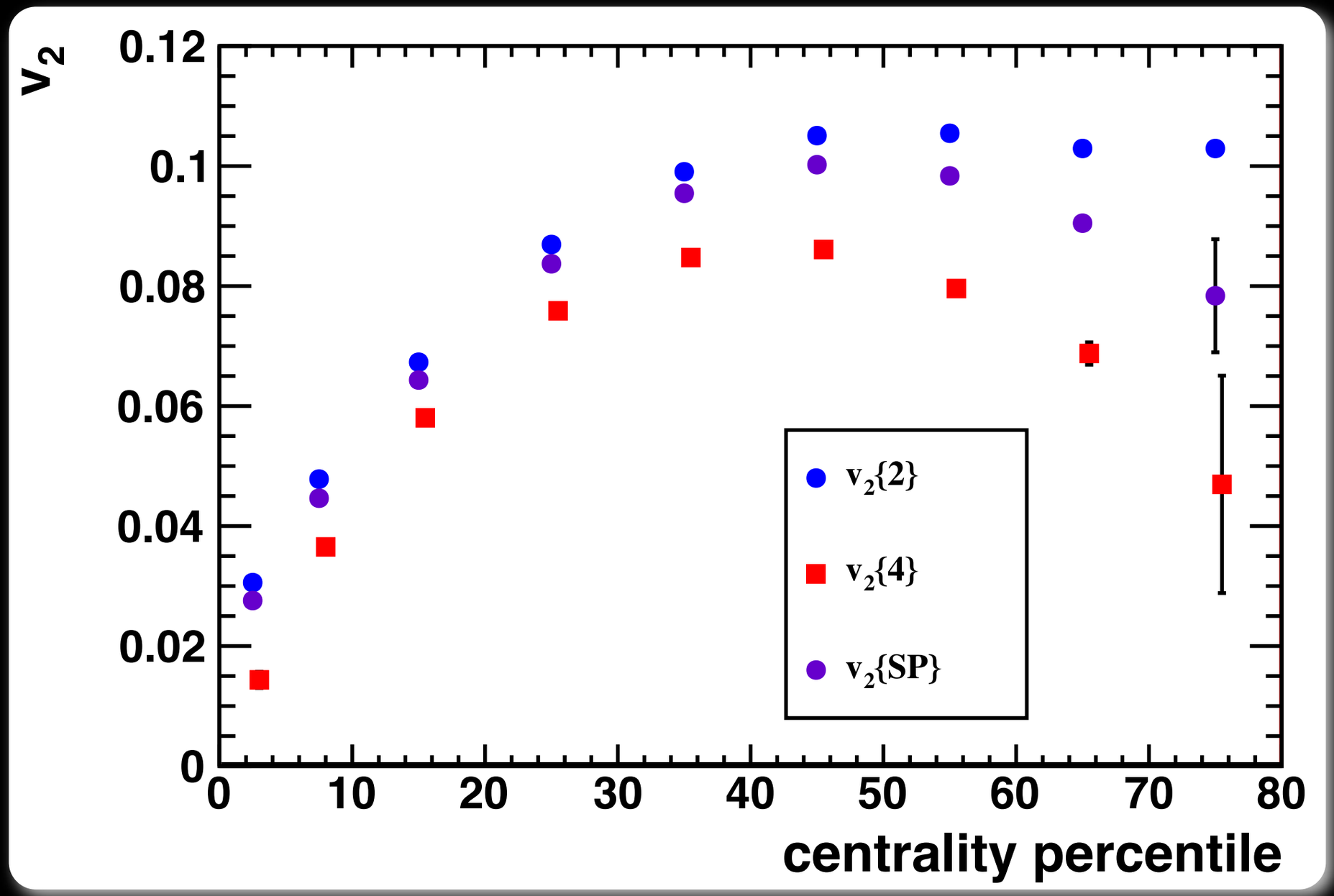
- Methods behave differently (not a bad thing because we understand!)
- two particle methods (including event plane method) are very sensitive to nonflow
- all methods are effected by event-by-event fluctuations of the flow

- if there are no fluctuations and no nonflow all methods give the same answer
- if there is only nonflow the multi-particle methods correct for the bias
- if there are only fluctuations the true flow in exactly in between the $v_2\{2\}$ and $v_2\{4\}$
  - we thus have measured the mean and the fluctuations!
- in reality we have fluctuations and nonflow and then $v_2\{4\}$ gives a lower bound but the difference between $v_2\{2\}$ and $v_2\{4\}$ is due to σ and δ to remove δ from $v_2\{2\}$ one has to do some less controlled tricks

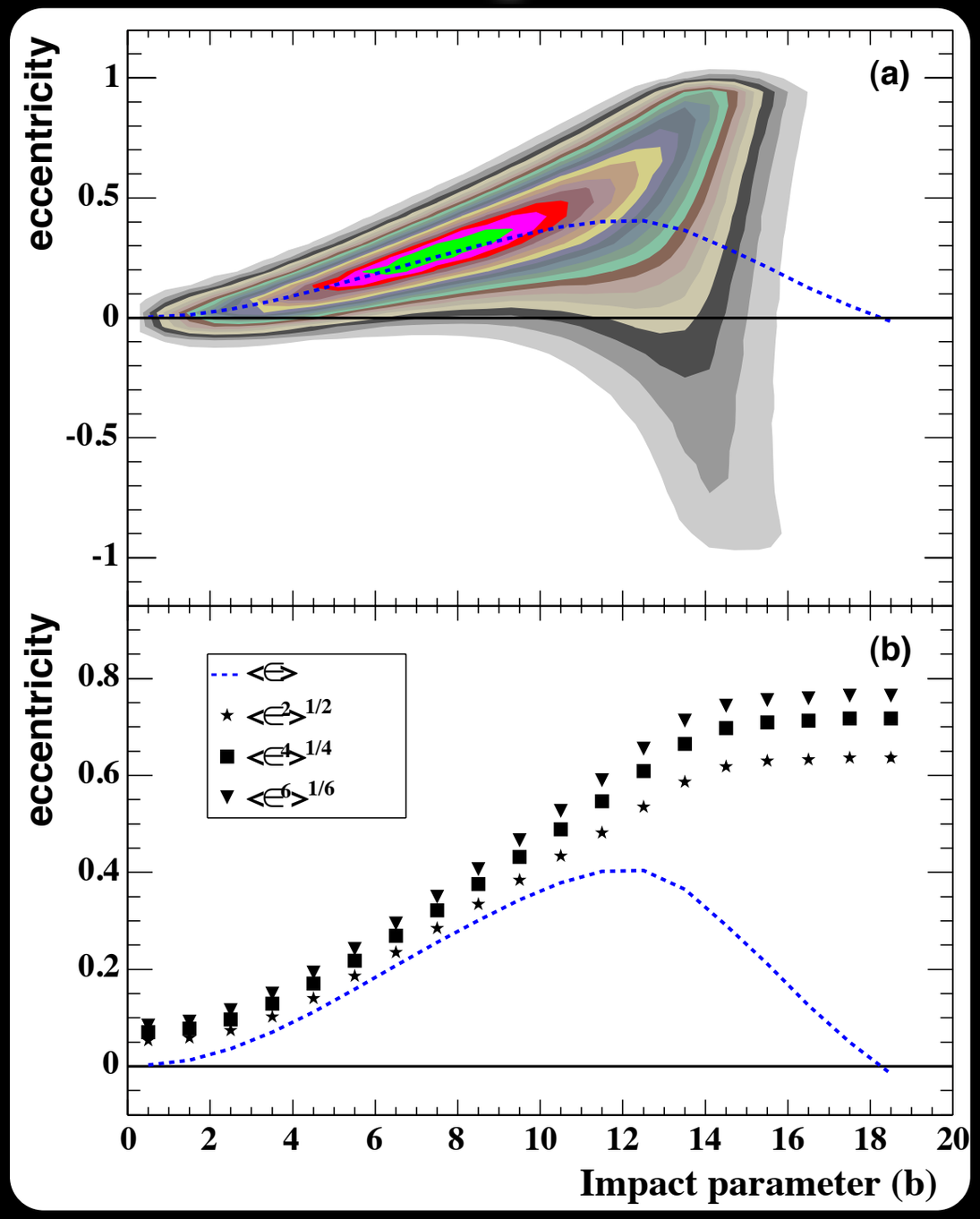
estimate δ


$v_2\{SP\}$
0.12
0.1
0.08
0.06
0.04
0.02
0
10
20
30
40
50
60
70
80
90
centrality percentile
Reference flow
$|\Delta\eta| = 0.0$
$|\Delta\eta| = 0.2$
$|\Delta\eta| = 0.4$
$|\Delta\eta| = 0.6$
$|\Delta\eta| = 0.8$
$|\Delta\eta| = 1.0$ (default)

estimate δ and σ



Why does v_2 fluctuate?



M. Miller and RS, arXiv:nucl-ex/0312008

- measured: $v_2\{2\} = \sqrt{(\langle v_2\rangle^2 + \sigma_v^2 + \delta)}$
- using: $v_2 \propto \varepsilon$
- If the eccentricity fluctuates

$$\langle \varepsilon^2 \rangle - \langle \varepsilon \rangle^2 \neq 0$$

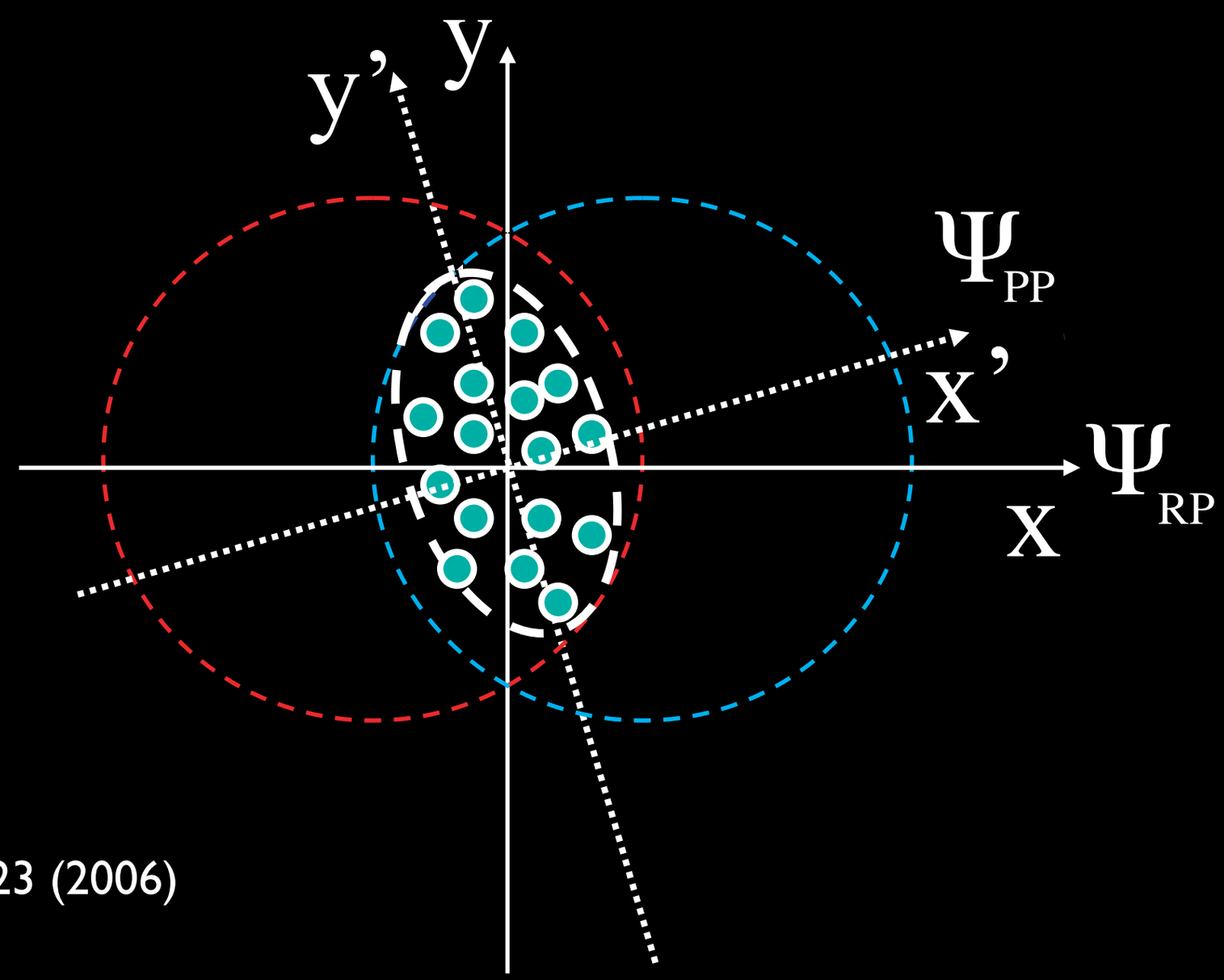
$$\langle v_2 \rangle \neq \sqrt{\langle (v_2)^2 \rangle}$$

- fluctuations can change our estimate of v_2!

eccentricity fluctuations:
M. Miller and RS, arXiv:nucl-ex/0312008 (2003)
participant eccentricity
PHOBOS QM2005: Nucl. Phys. A774: 523 (2006)

Fluctuations and Planes

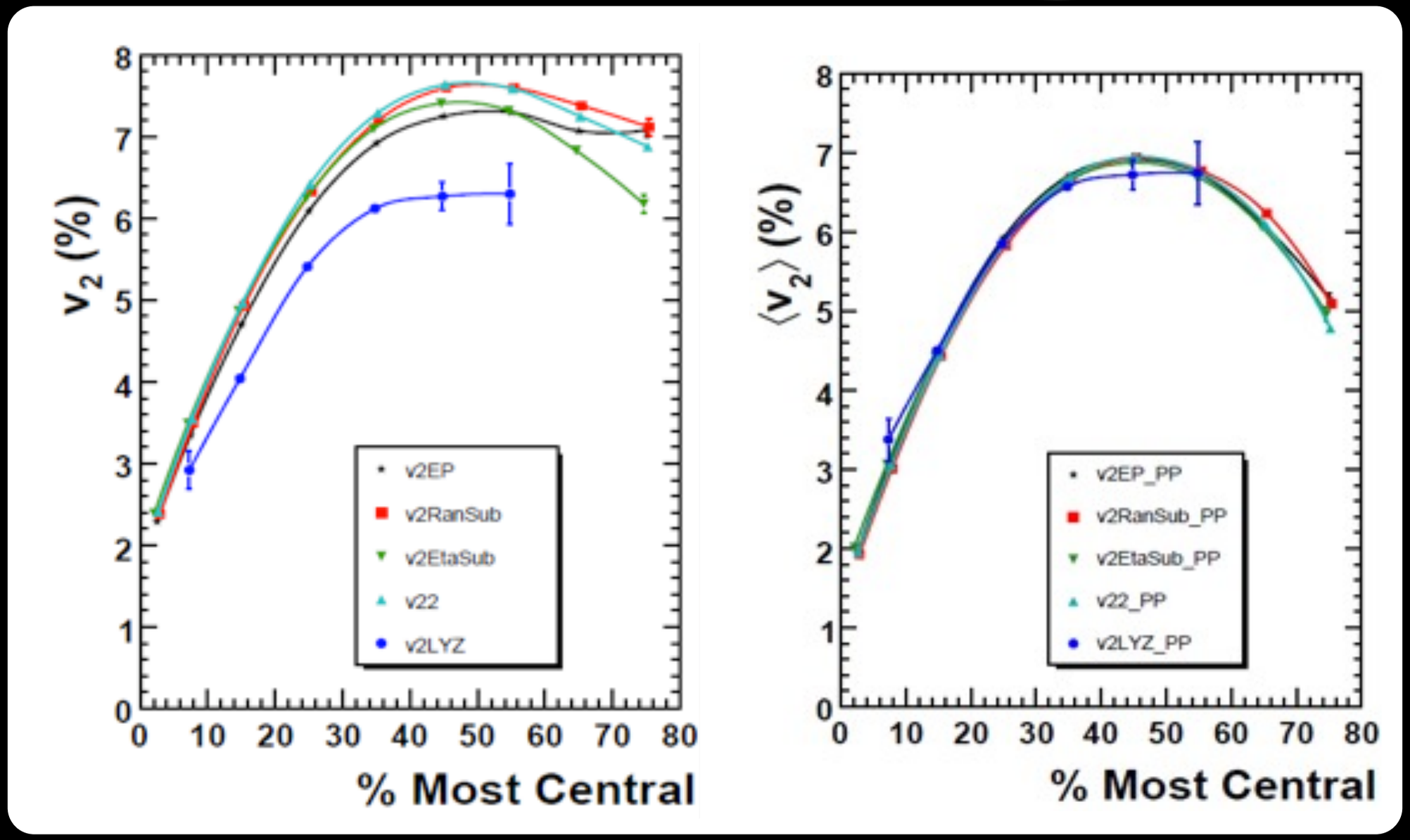
- RP the reaction plane, defined by the impact parameter
- PP the participant plane, defined by the major axis of the created system



PHOBOS QM2005: Nucl. Phys. A774: 523 (2006)

- Fluctuations change the angle of the symmetry plane

with some assumptions



assume fluctuations come from MC Glauber participant eccentricity and nonflow is proportional to nonflow measured in pp diluted by N_{part}

arXiv:0809.2949 [nucl-ex]

Collective phenomena in non-central nuclear collisions

October 6, 2008 Draft

Sergei A. Voloshin, Arthur M. Poskanzer, and Raimond Snellings

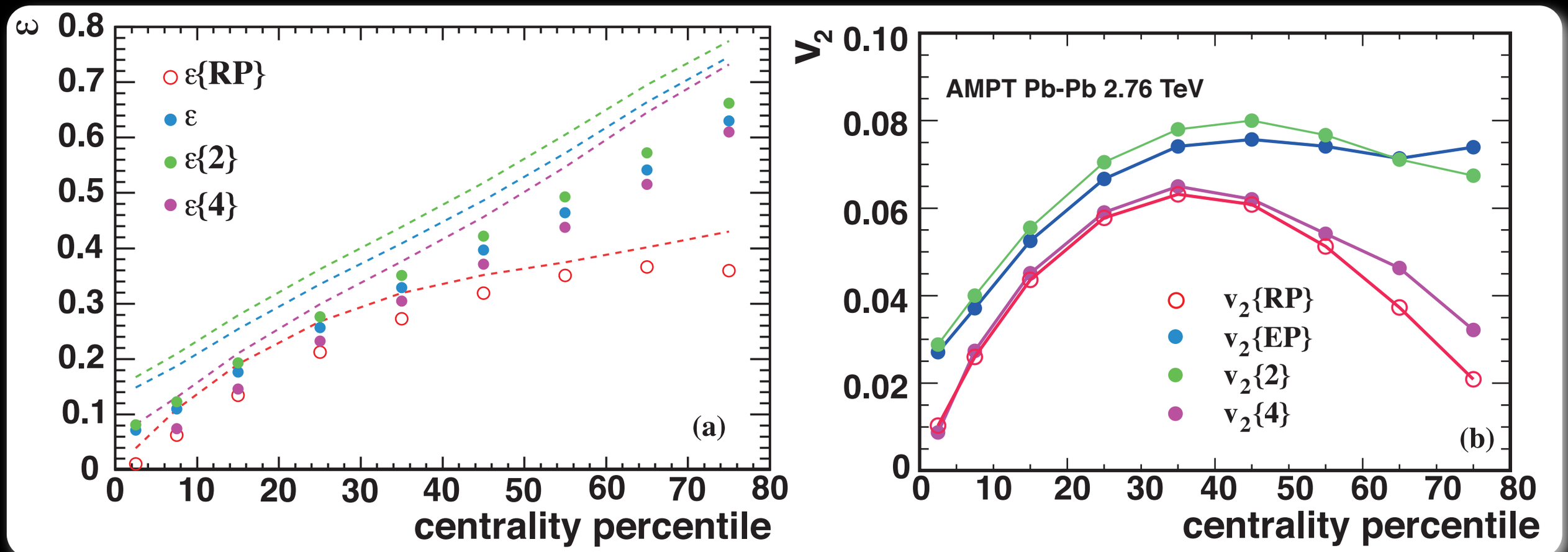


Abstract Recent developments in the field of anisotropic flow in nuclear collision are reviewed. The results from the top AGS energy to the top RHIC energy are discussed with emphasis on techniques, interpretation, and uncertainties in the measurements.

The End

Fluctuations and Methods

dashed lines are for N_{coll}, markers for $N_{wounded}$



multi-particle methods measure flow in the reaction plane while two particle methods measure in the participant plane + fluctuations + non-flow

R.S. Bhalerao , J-Y. Ollitrault Phys.Lett.B641:260-264,2006, S.A. Voloshin, A.M. Poskanzer, A. Tang, G. Wang Phys.Lett.B659:537-541,2008

Flow Analysis Methods

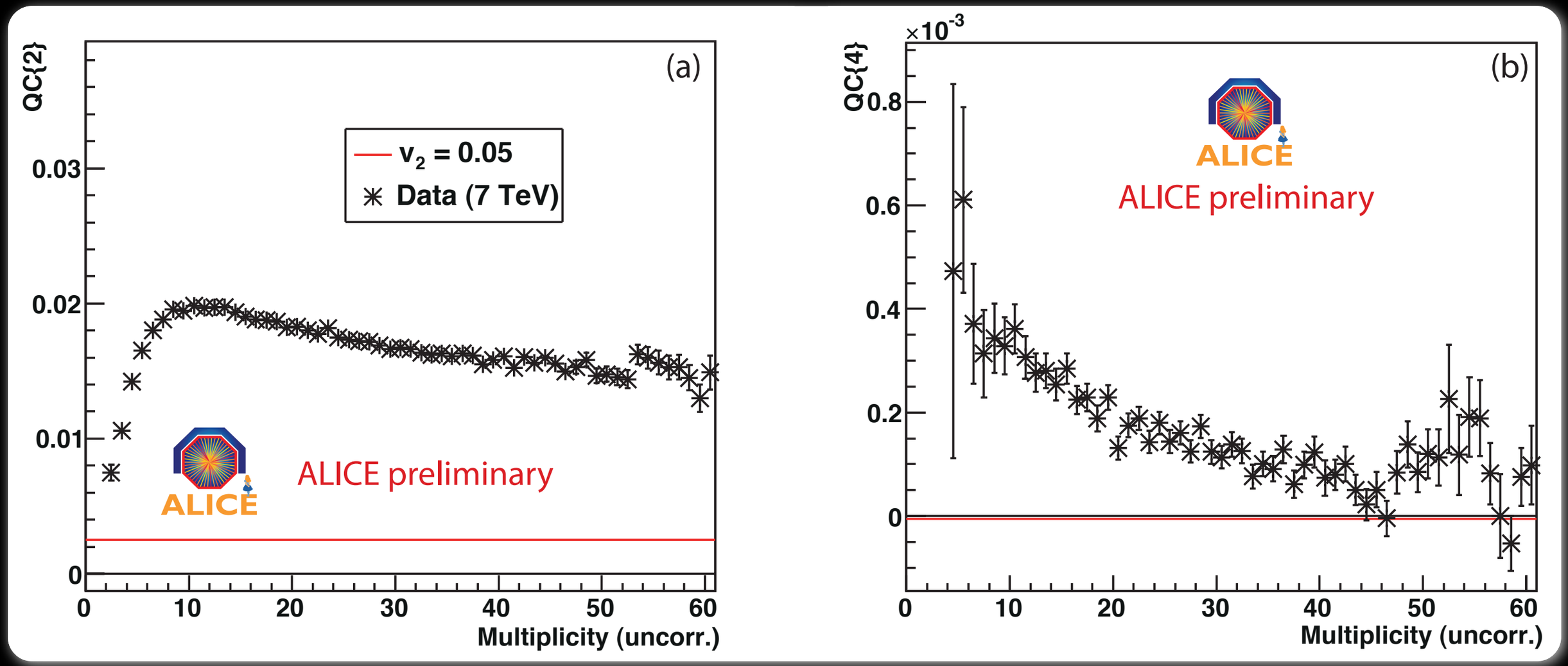
name	
MCEP	versus true reaction plane
SP	2 particle
GFC	2 and multi-particle cumulants
QC	2 and multi-particle cumulants
FQD	full event flow vector
LYZ	full event flow vector

methods have different sensitivity to nonflow and fluctuations
For flow analysis in ALICE we use and compare all of them

QC is a new approach to calculate the cumulants without using numerical solutions

A.Bilandzic, R.S, S. Voloshin, *"Flow analysis with cumulants: direct calculations"*, arXiv: 1010.0233 [nucl-ex]

What about p-p?

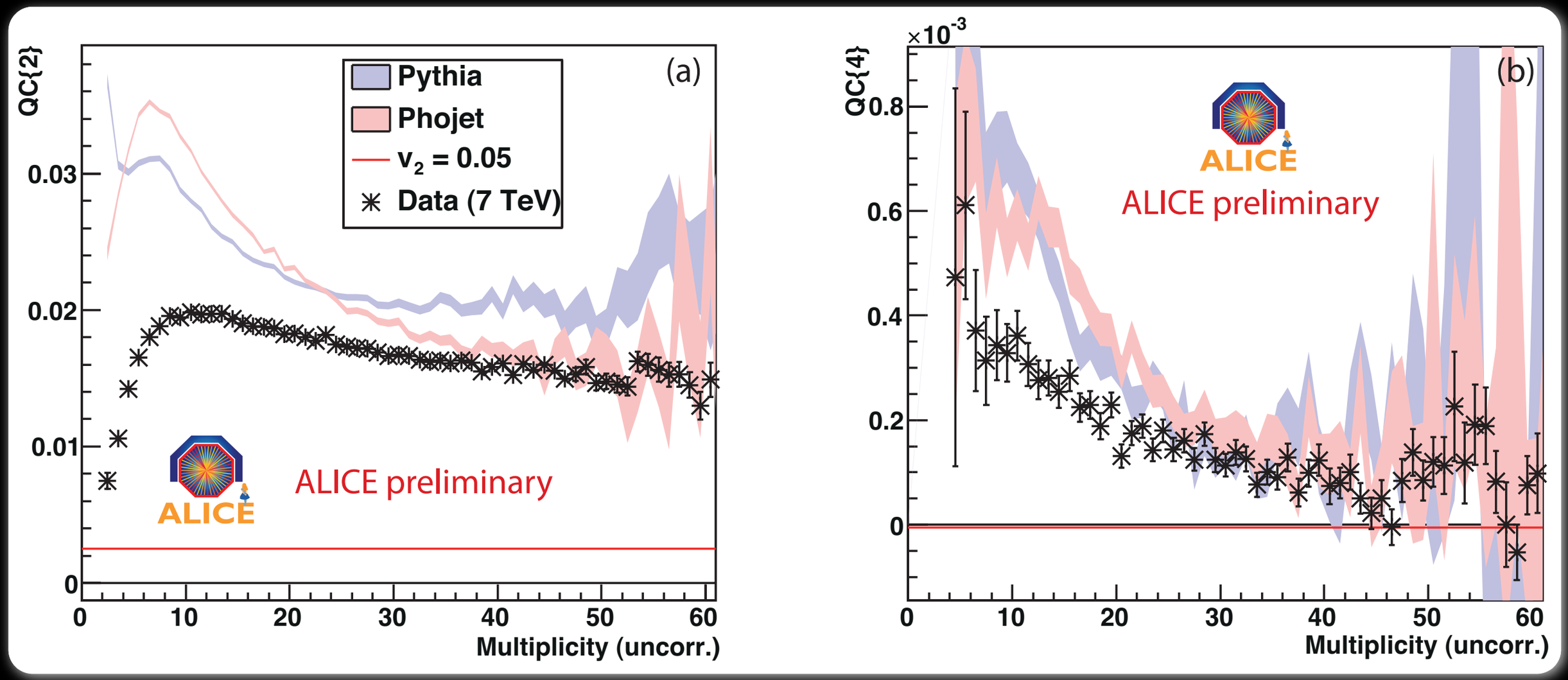


remember $QC\{2\} = v^2$, $QC\{4\} = -v^4$

the 2 and 4-particle correlations decrease with increasing number of particles produced which is typical behavior of correlations involving only few particles

the 4-particle cumulant, QC{4}, even has the wrong sign compared to true flow!

What about p-p?

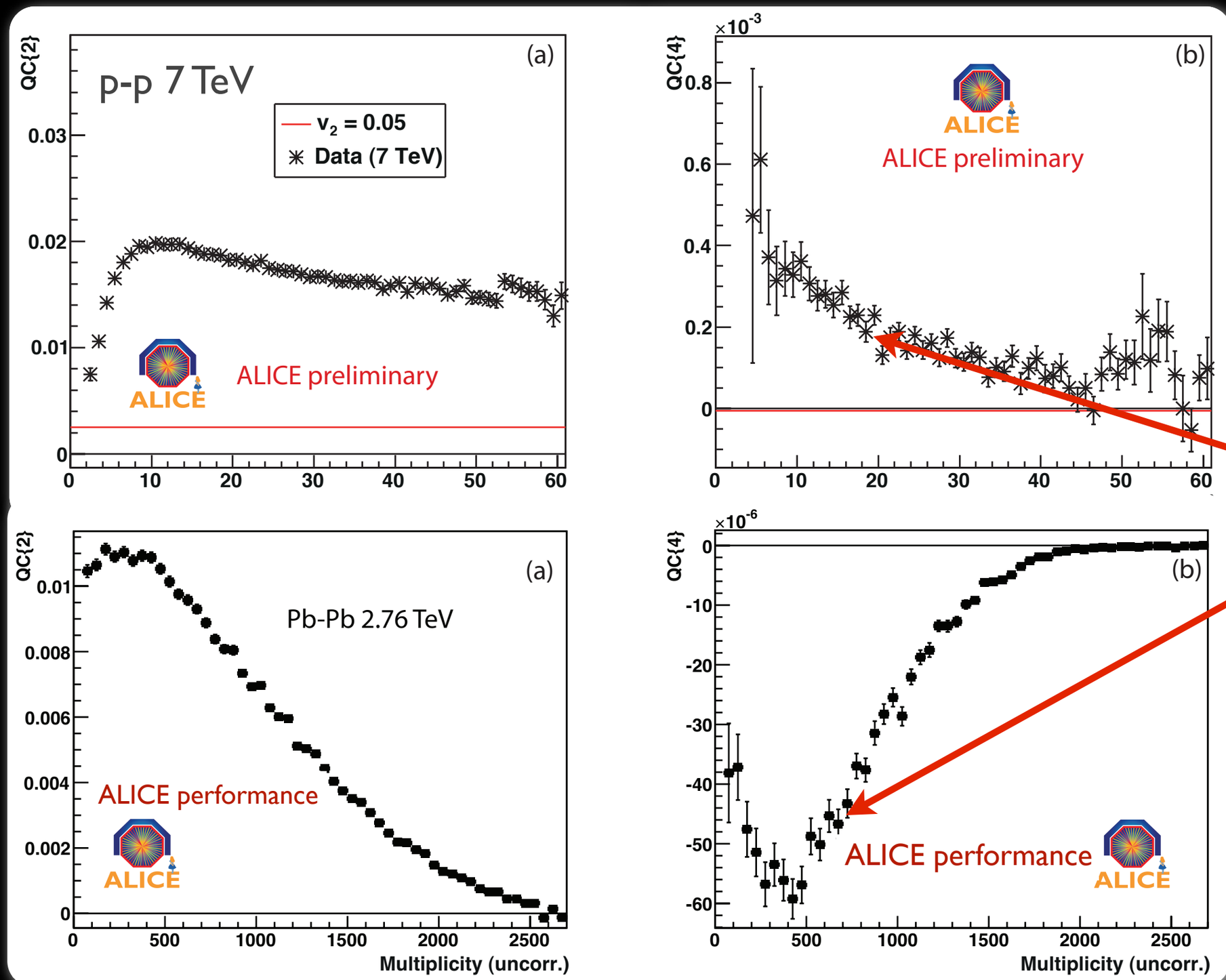


remember $QC\{2\} = v^2$, $QC\{4\} = -v^4$

models like PYTHIA and PHOJET have the correct sign for the correlations and capture the general trends observed in p-p

no evidence for elliptic flow in this multiplicity range

Cumulants in Pb-Pb



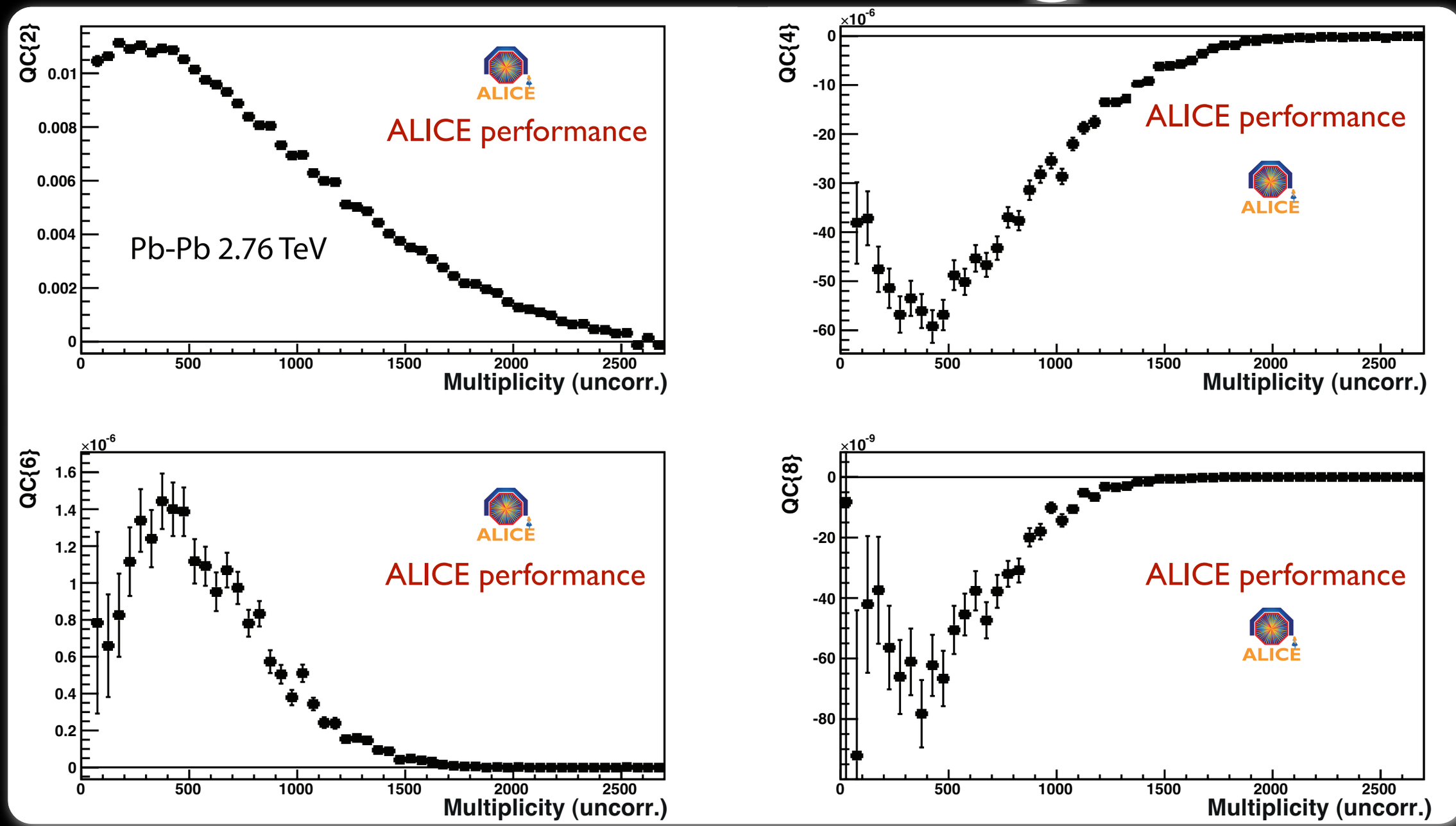
p-p

note the change in sign from p-p to Pb-Pb

Pb-Pb

remember $QC\{2\} = v^2$, $QC\{4\} = -v^4$

Flow at first sight!

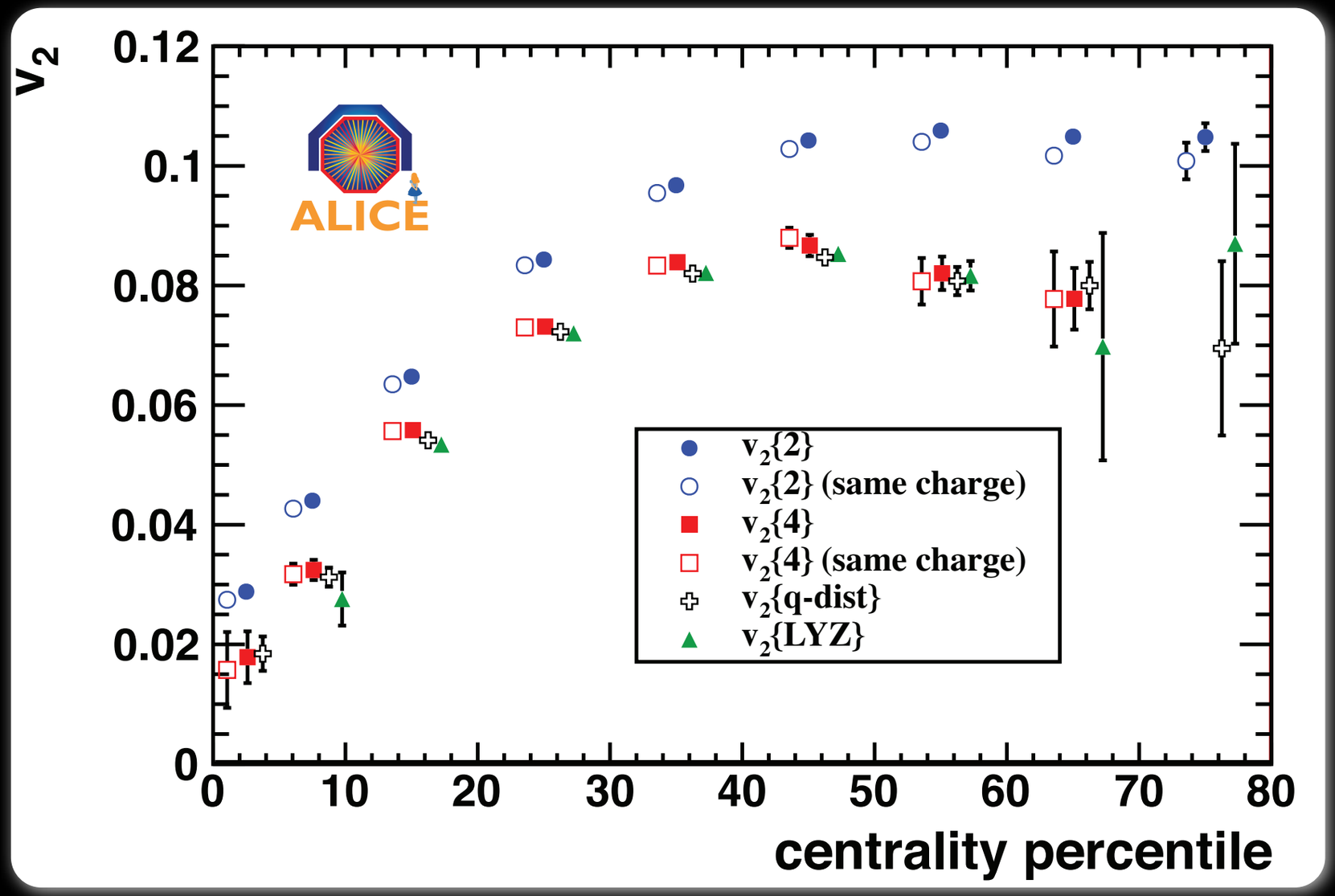


remember $QC\{2\} = v^2$, $QC\{4\} = -v^4$, $QC\{6\} = 4v^6$. and $QC\{8\} = -33v^8$

cumulants show strong collective flow!

v_2 in ALICE

K. Aamodt et al. (ALICE Collaboration)
PRL 105, 252302 (2010)



expected difference between two and multi-particle estimates

multi-particle estimates agree within uncertainties as is expected for collective flow!